MASS 2023 Course:
Gravitation and Cosmology
(Syllabus)

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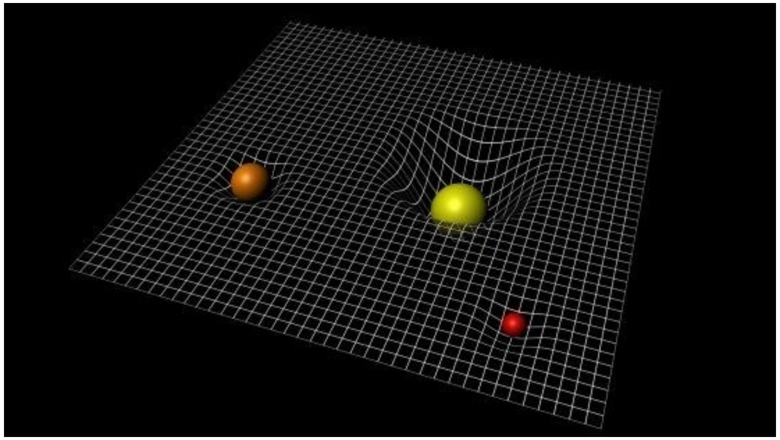
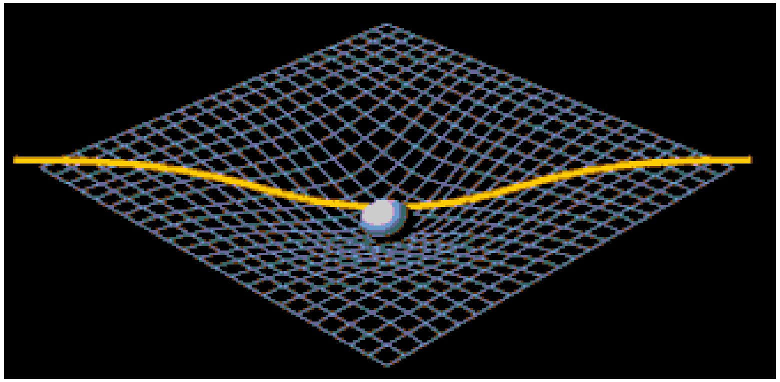
Lectures 1

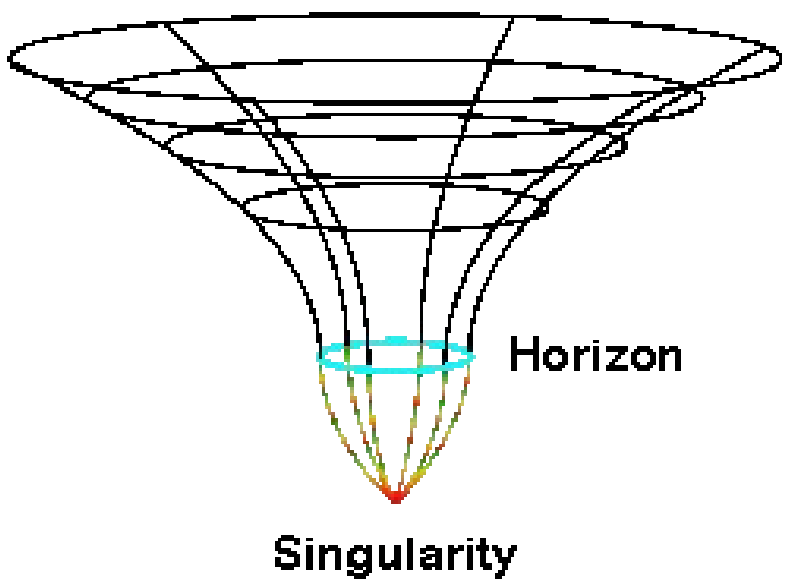
1. General relativity (GR) as geometric theory of gravitation:
   - Basic mathematical concepts: spacetime as 4-dimensional pseudo-Riemannian manifold. Curved spacetime and curvilinear coordinates, covariant and contravariant transformation rules. Scalars (invariants), vectors and tensors. Metric tensor and tensor algebra.
   - Spacetime in absence of gravity: basics of Special Relativity, Minkowski spacetime and Lorentz transformations.
   - Introduction to GR: Principle of general covariance and equivalence principle. Affine connections and Christoffel symbols. Geodesic equations. Newtonian limit. Covariant differentiation. Riemann-Christoffel curvature tensor and Bianchi identities. Ricci curvature tensor and Ricci scalar. Einstein tensor. Parallel transport. Minimal-coupling principle. Stress-energy tensor as source of gravity. Perfect fluid.

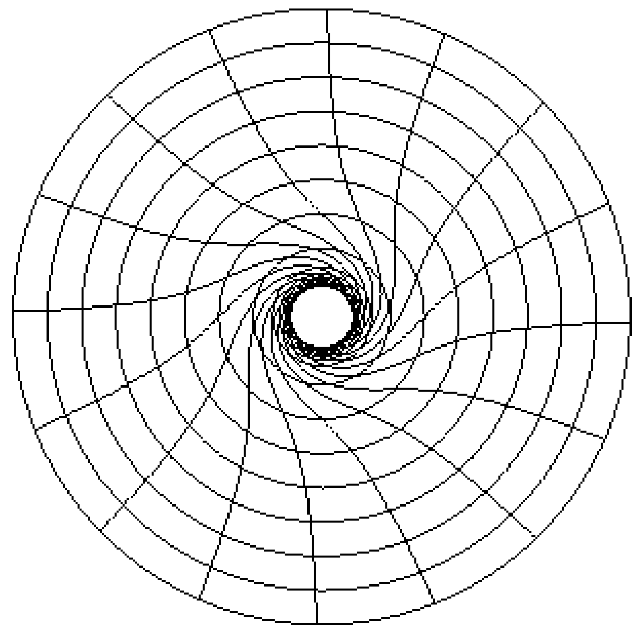




Lectures 2

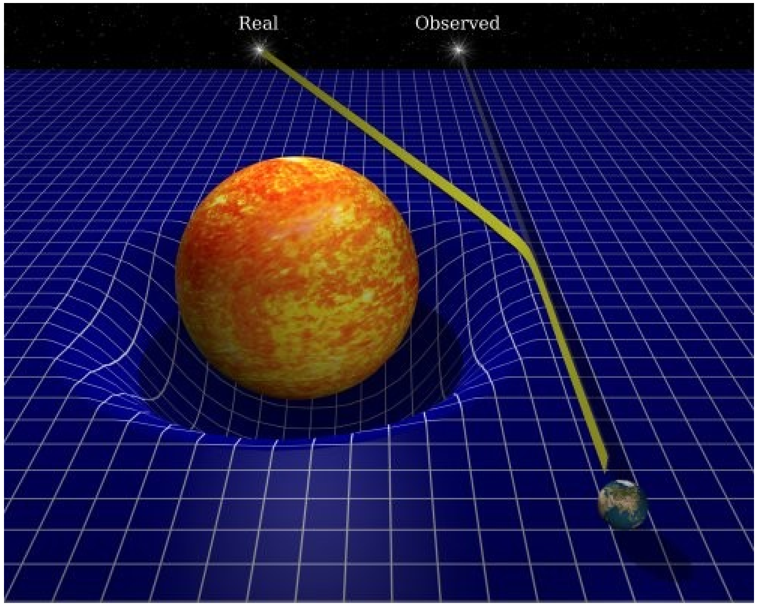
- Calculus of variations and variational principle: Lagrangian. Euler-Lagrange equations. Hamilton's principle of stationary action.
- Einstein field equations (EFE): Einstein-Hilbert action. Cosmological constant. EFE in vacuum and in the presence of matter.
- Vacuum solutions to EFE: Schwarzschild metric, Kerr metric, Reissner-Nordström metric, Kerr-Newman metric
- Black holes

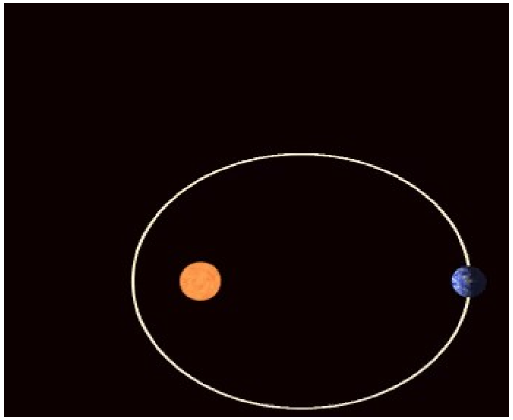


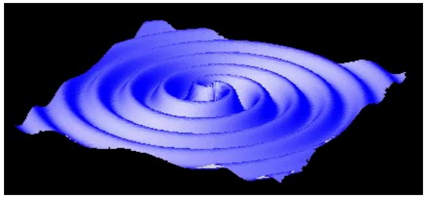


Lectures 3: Experimental tests of GR

1. Four classic Solar System tests of GR:
   - Perihelion precession of Mercury's orbit
   - Deflection of light by the Sun
   - Gravitational redshift of light
   - Gravitational (Shapiro) time delay
2. Other experimental tests of GR:
   - Precession of orbiting gyroscopes (Lense-Thirring effect)
   - Discovery of the central supermassive black hole (SMBH) of the Milky Way
   - Observed images of the SMBH shadows at the centers of M87 and our Galaxy
   - Relativistically broadened spectral lines from Active Galactic Nuclei (AGN)
   - Gravitational waves and their detection





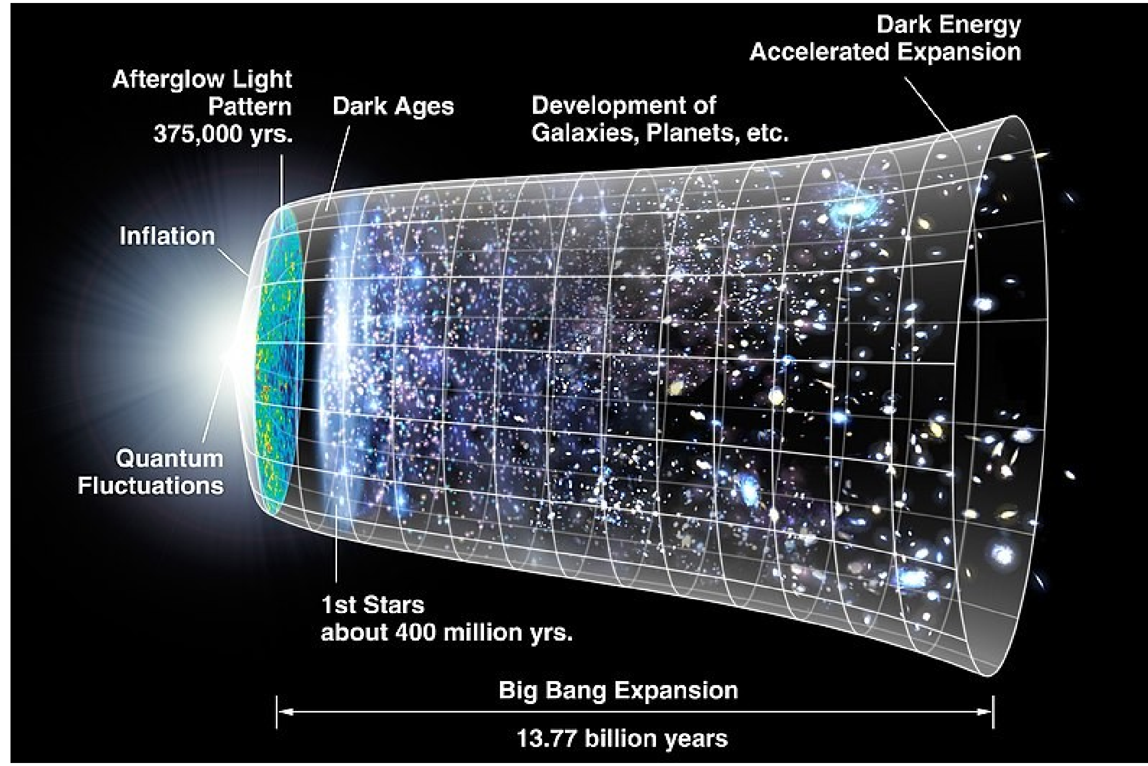


Lectures 4

2. Cosmology based on GR:

- Big Bang theory: stages in cosmic evolution.

- Cosmological principle: homogeneity and isotropy of the universe. Construction of Friedmann-Lemaître-Robertson-Walker (FLRW) metric.

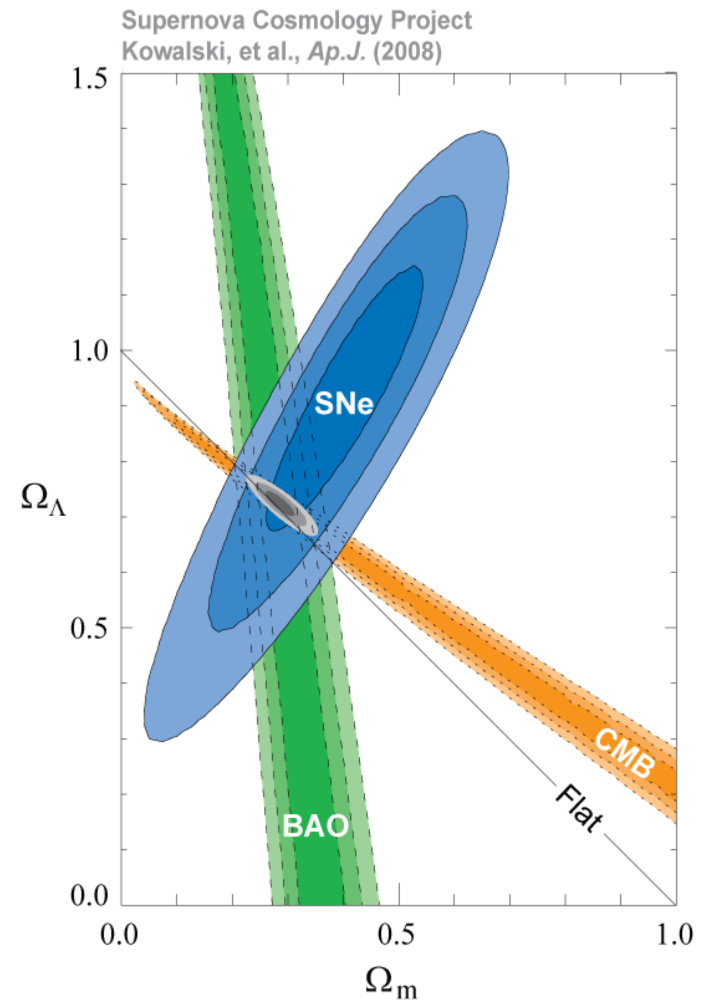


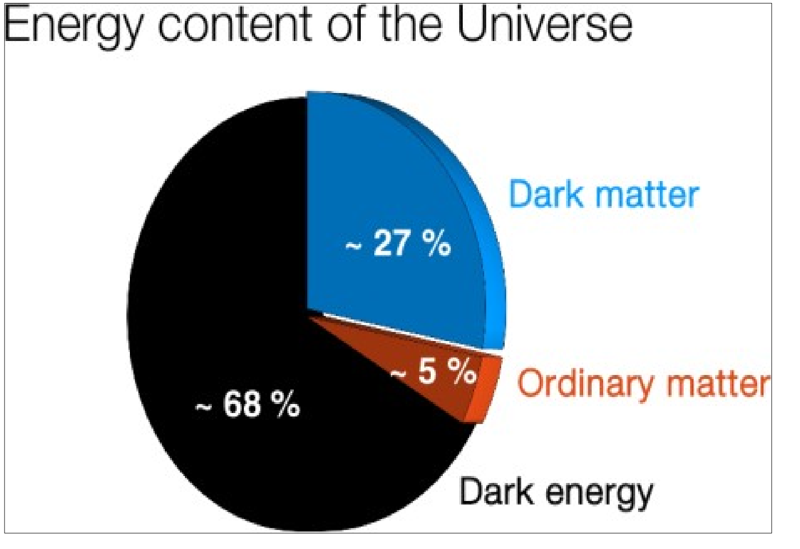
- Cosmic scale factor: cosmological redshift. Hubble parameter. Deceleration parameter.

- Friedmann equations (FE): derivation from EFE for FLRW metric and perfect fluid. Cosmological density parameters. Mass-energy budget of the universe (baryonic matter, dark matter and dark energy).

Lectures 5

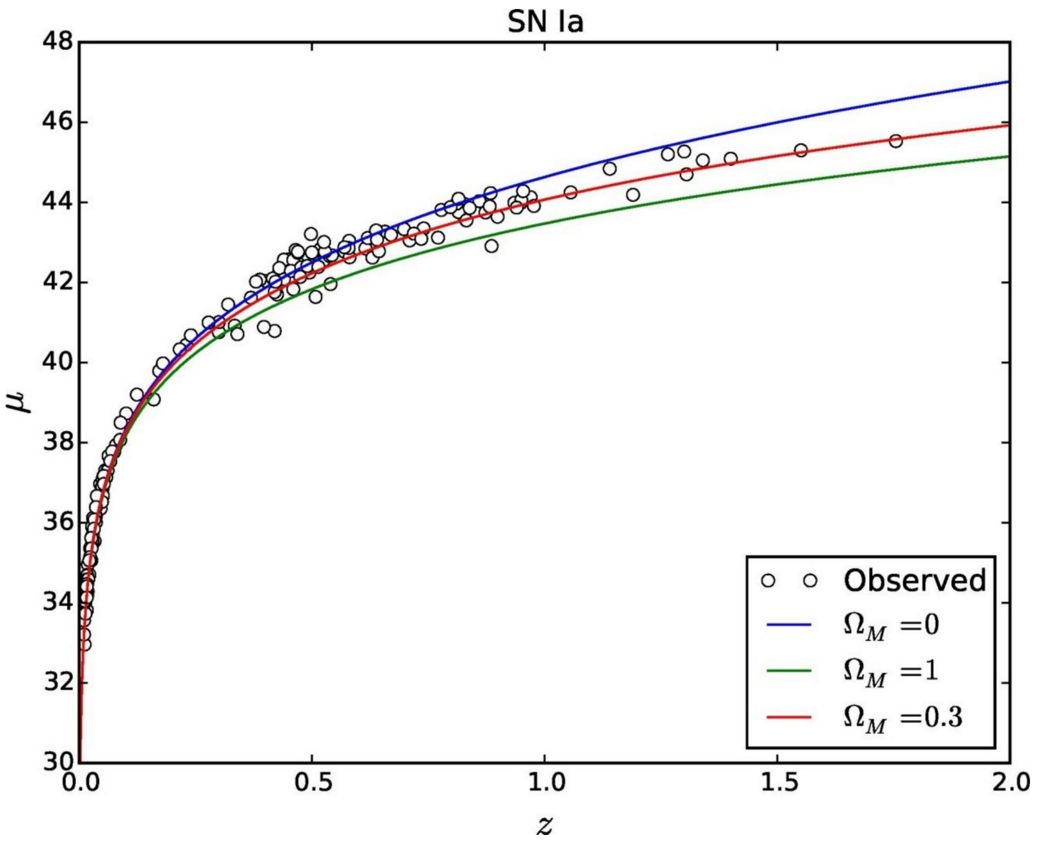
- Models of Friedmannian cosmology: Friedmann-Einstein, Einstein-de Sitter, de Sitter and standard ΛCDM cosmological model.
- Observational cosmology: comoving coordinates and cosmological distances (proper, comoving, luminosity and angular diameter distance)
- Hubble-Lemaître law
- Supernovae of type Ia (SN Ia)
- Cosmic microwave background radiation (CMBR)
- Baryon acoustic oscillations (BAO)





Exercises

1. Solving problems on the blackboard (both GR and cosmology)
2. Comparisons of theoretical models with astronomical observations (cosmology):
   - Writing scripts in Python programming language
   - You need to have installed Python 3 interpreter with NumPy, SciPy, Astropy and MatPlotLib/Pylab libraries



Final exam

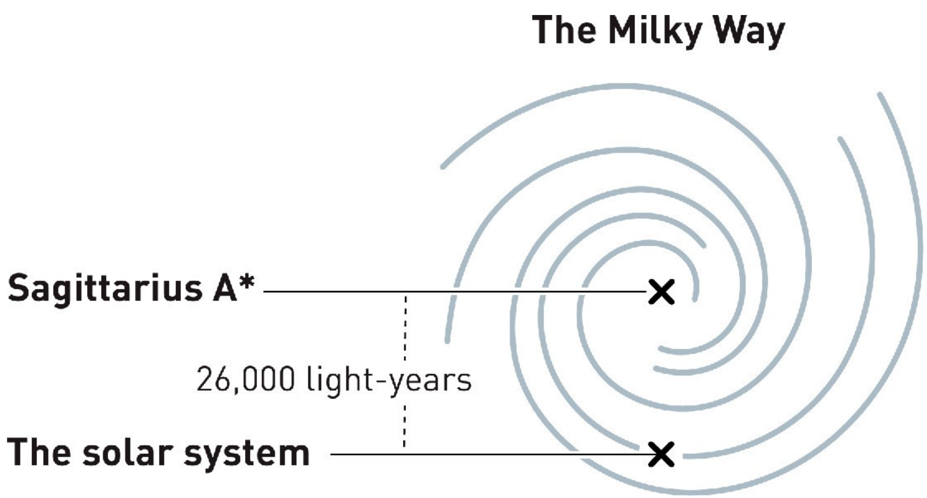
- First half of June
- Test will have two parts:
  - written (3 problems to solve)
  - oral (2 questions to answer)
- Problems and questions will cover both general relativity and cosmology, and each of them will bring maximum of 20 points

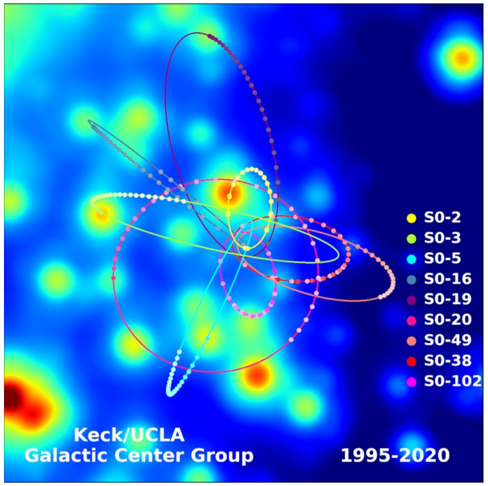
Points	Grade
51-60	6
61-70	7
71-80	8
81-90	9
91-100	10

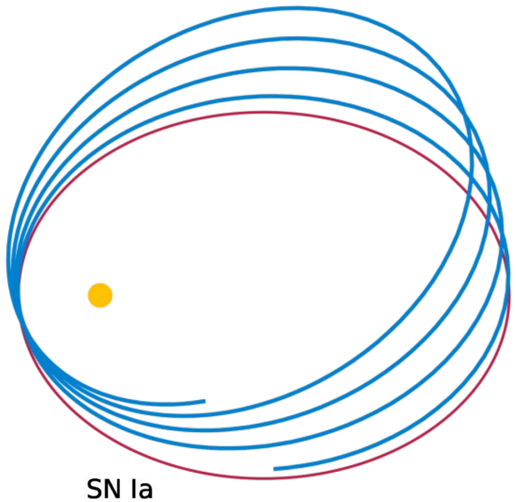
5 - failed
6 - satisfactory
7 - good
8 - very good
9 - excellent
10 - remarkable

Topics for master theses

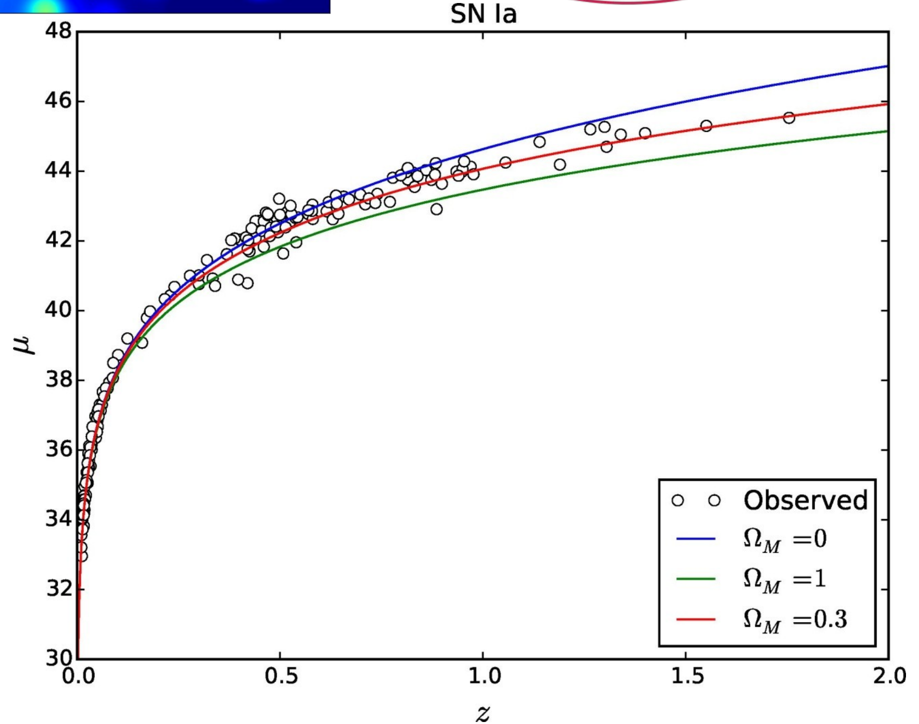
1. **General relativity:** "*Schwarzschild and Lense-Thirring precession of the bright S-stars orbiting around the supermassive black hole Sgr A* at the Galactic Center*"







2. **Observational cosmology:** "*Constraining the cosmological parameters by Hubble diagrams of type Ia supernovae (SN Ia)*"



Literature (textbooks)

- Sean M. Carroll, 1997. Lecture Notes on General Relativity, arXiv:gr-qc/9712019
- Weinberg, S., 1972, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, Wiley-VCH
- Weinberg, S., 2008, Cosmology, Oxford University Press Inc., New York, USA
- Peebles, P.J.E., 1993, Principles Of Physical Cosmology, Princeton University Press, Princeton, New Jersey, USA

Literature (recommended readings)

- Davis, T. M., Lineweaver, C. H. 2004, Expanding Confusion: Common Misconceptions of Cosmological Horizons and the Superluminal Expansion of the Universe, PASA, 21, 97 (arXiv:astro-ph/0310808)
- Hogg, D. W., 2000, Distance measures in cosmology, arXiv:astro-ph/9905116

Lectures, exercises and main literature will be available at:
http://pjovanovic.aob.rs/gc2023/