

MASS 2026 Course:
Gravitation and Cosmology

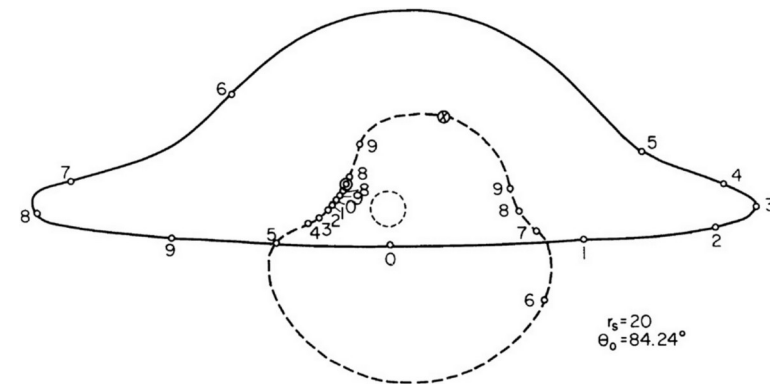
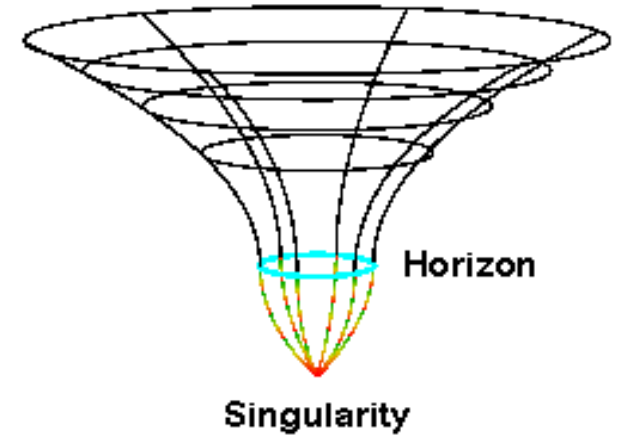
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Lecture 08

- More recent experimental tests of GR (because of these successes, GR is considered as modern and **standard theory of gravity**)
- Black holes:
 - Single and binary SMBHs at the centers of Active Galactic Nuclei (AGN)
 - Relativistically broadened spectral lines from AGN
 - Discovery of the central supermassive black hole (SMBH) of the Milky Way
 - Observed images of the SMBH shadows at the centers of M87 and our Galaxy
- Gravitational waves and their detection
- Exercise

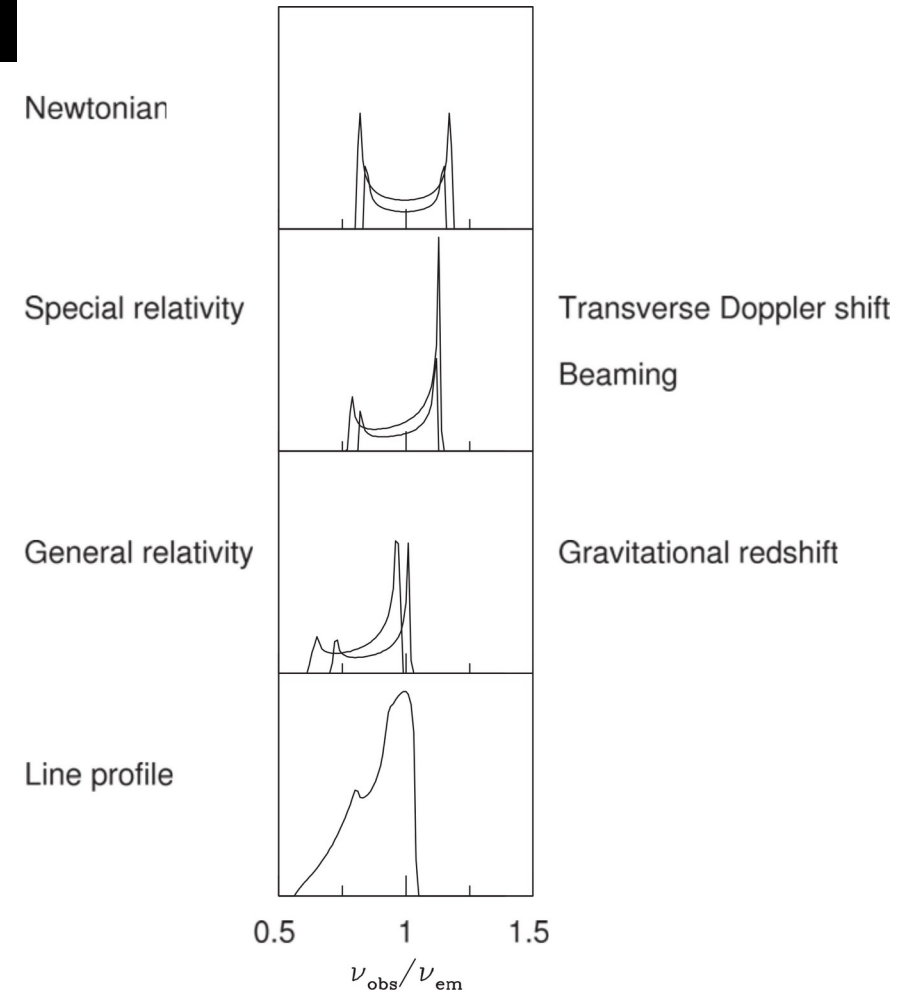
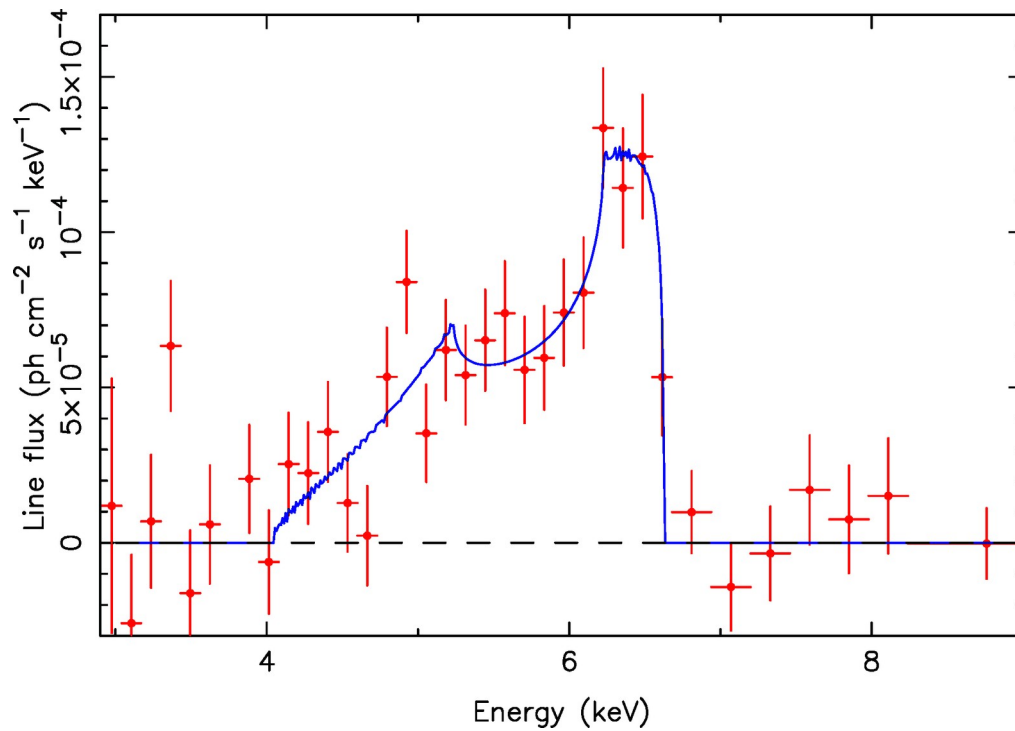
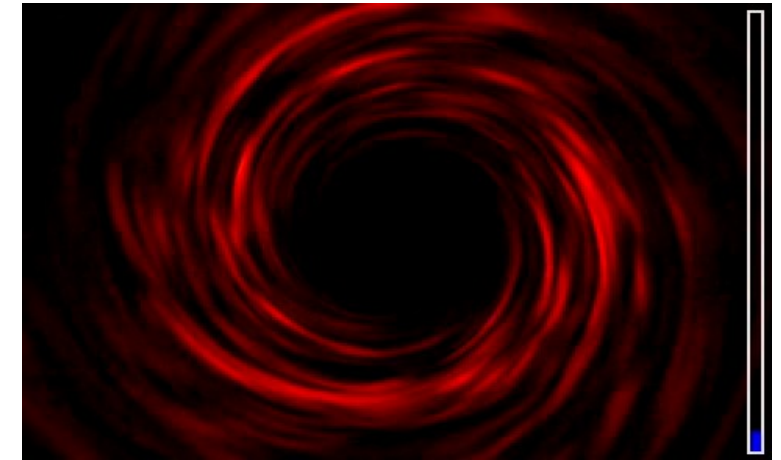
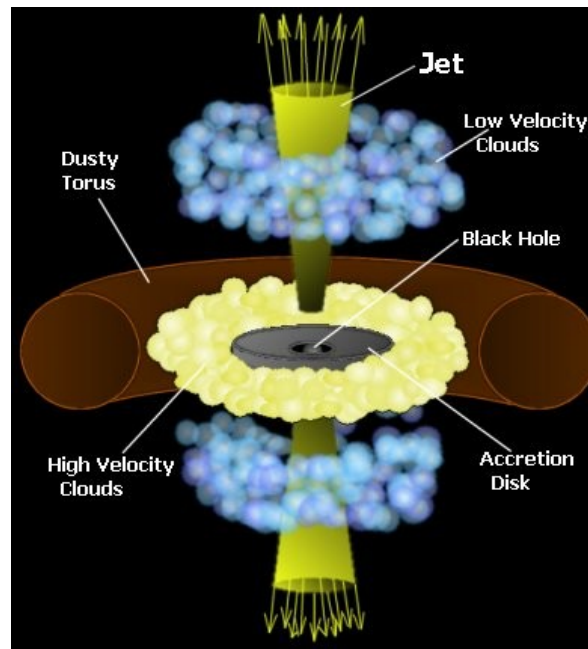
Black holes

- **Black hole** is a region of spacetime where gravity is so strong that nothing, not even light, can escape its event horizon
- Only three measurable parameters (not including the Hawking temperature): **mass**, angular momentum (**spin**) and **charge**
- Classification according to the **metric**:
 1. **Schwarzschild** (non-rotating and uncharged)
 2. **Kerr** (rotating and uncharged)
 3. **Reissner–Nordström** (non-rotating and charged)
 4. **Kerr–Newman** (rotating and charged)
- Classification according to their **masses**:
 1. **Mini, micro or quantum mechanical**: $M_{BH} \ll M_{\odot}$
(primordial black holes in the early universe)
 2. **Stellar-mass**: $M_{BH} < 10^2 M_{\odot}$
(in the X-ray binary systems)
 3. **Intermediate-mass**: $M_{BH} \sim 10^2 - 10^5 M_{\odot}$
(in the centers of globular clusters)
 4. **Supermassive**: $M_{BH} \sim 10^5 - 10^{10} M_{\odot}$
(in the centers of all galaxies, including ours)
- Astrophysical black holes are gravitationally collapsed objects, and they are usually assumed to be electrically neutral



A circular orbit (solid line) of a point source in the equatorial plane around an extreme Kerr black hole

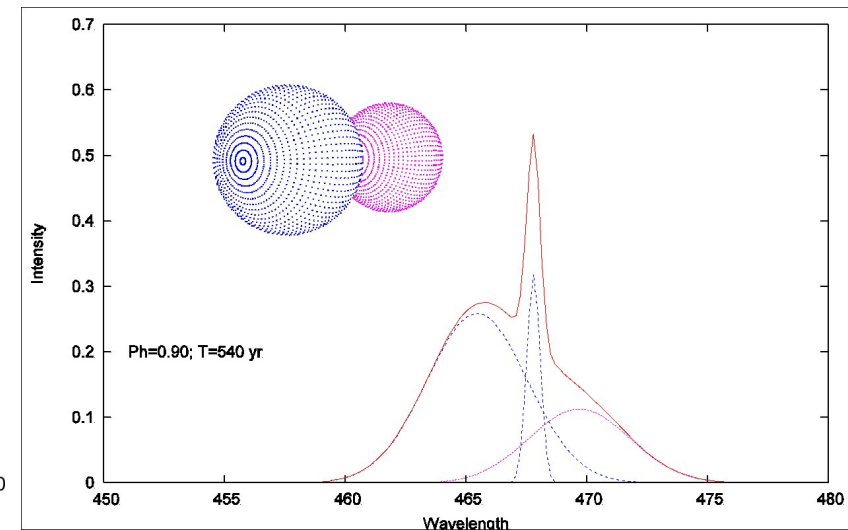
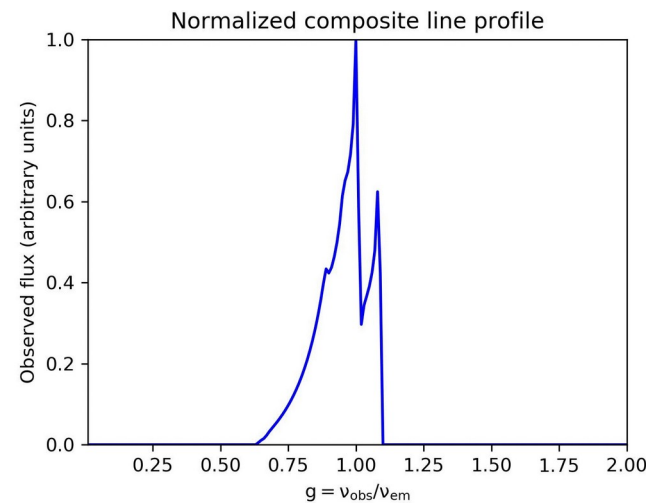
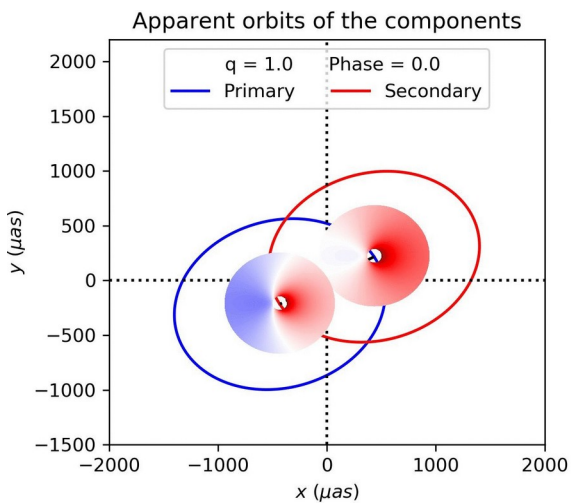
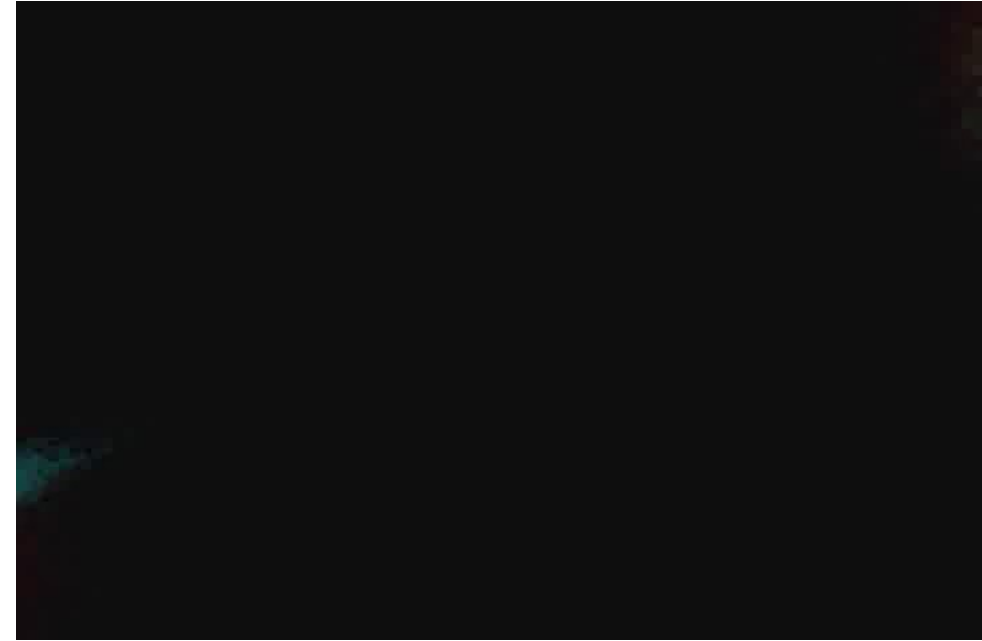
Single SMBHs at the centers of AGN and their relativistically broadened spectral lines



The observed Fe K α line profile from Seyfert I galaxy MCG-6-30-15, and the modeled profile from a relativistic accretion disk around a Schwarzschild SMBH

Supermassive black hole binaries (SMBHBs)

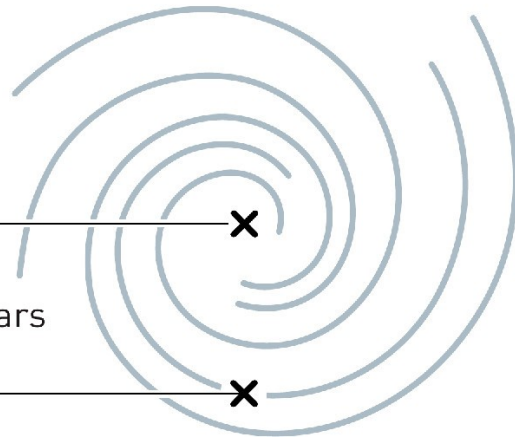
- SMBHB originate in galactic mergers and are expected to be present in the centers of a number of galaxies
- Their coalescences represent the strong emitters of low-frequency (nanohertz) gravitational waves (GWs)
- Direct observation of SMBHBs with current observation techniques is impossible
- AGN spectral line shapes in different bands can indicate the presence of SMBHBs
- Specific periodical variability in the optical and X-ray line profiles, depending on the orbital phase of the system, could represent the electromagnetic counterparts of GWs



Fe K α (top) and H β (right) line profiles emitted from a SMBHB

Sgr A*: central SMBH of the Milky Way

The Milky Way

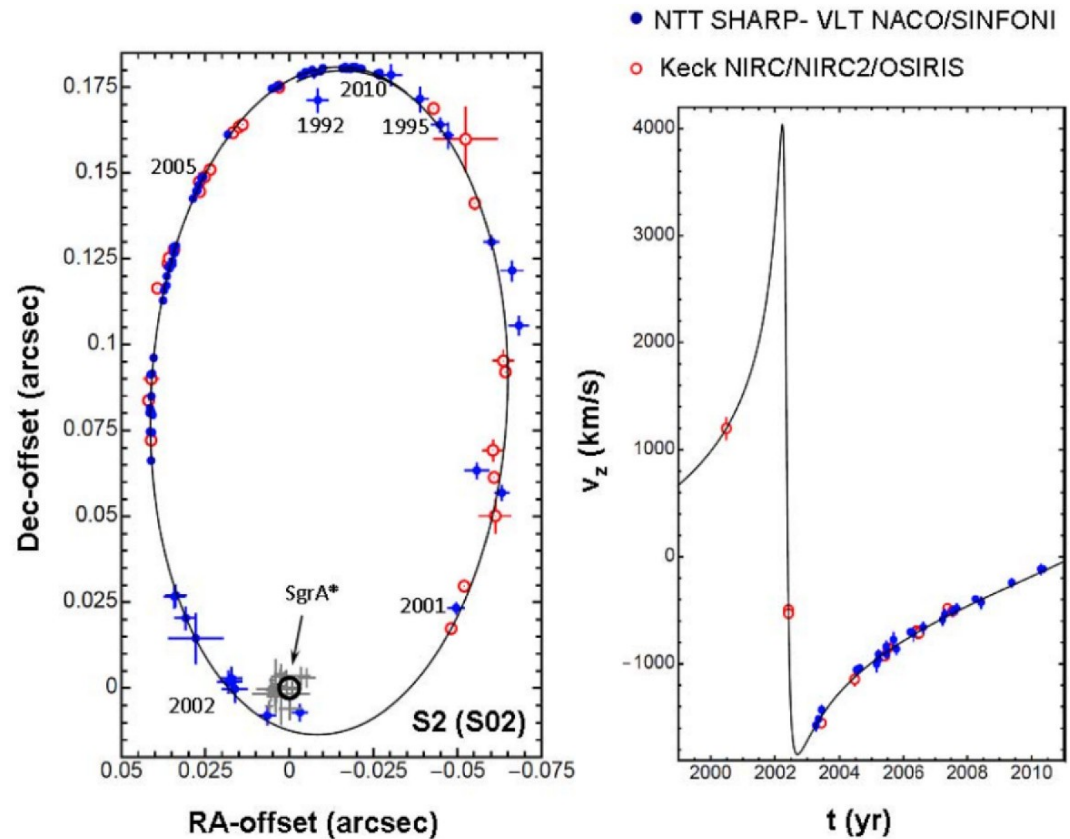
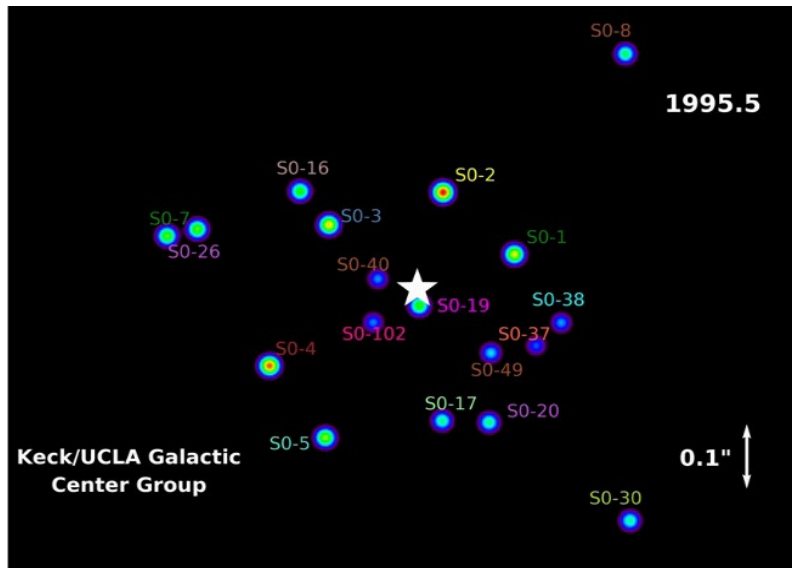


Sagittarius A*

26,000 light-years

The solar system

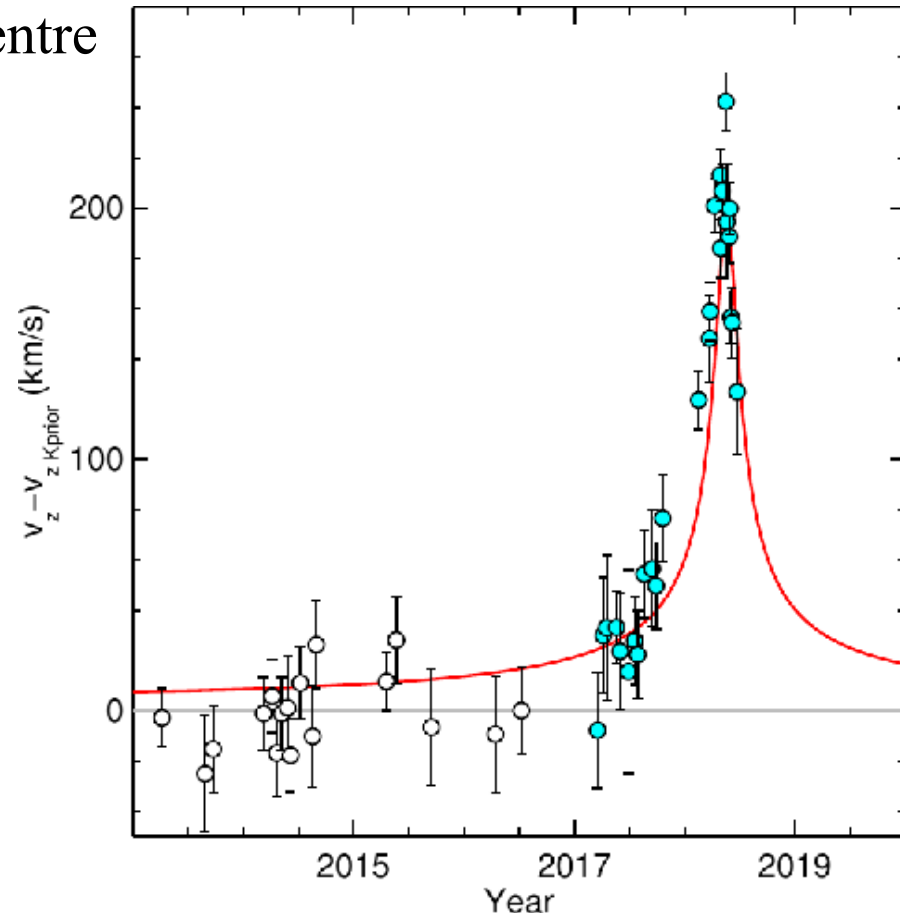
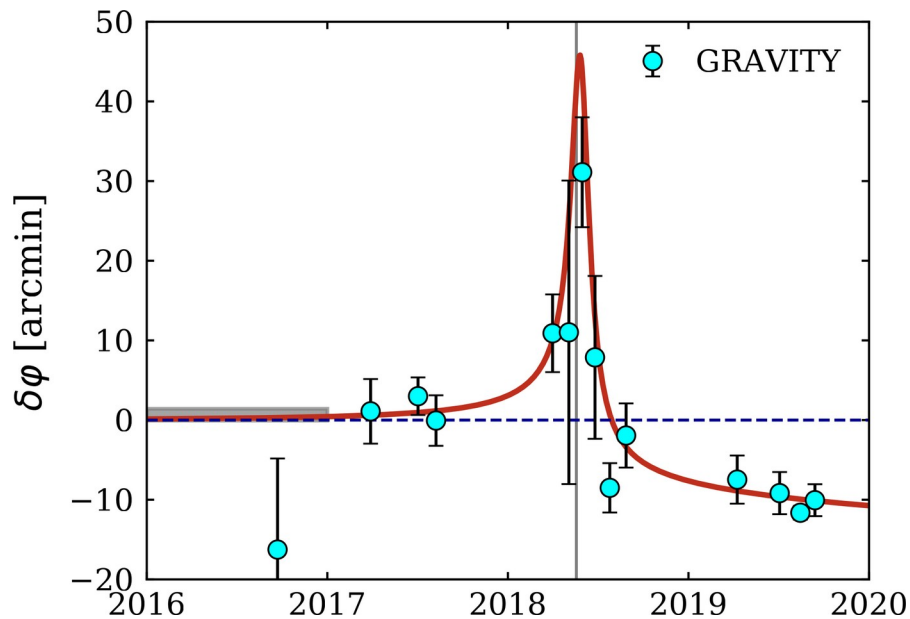
- Stellar orbits around Sgr A* monitored by these 2 groups:
 - R. Genzel (ESO): **New Technology Telescope & Very Large Telescope, Chile**
 - A. Ghez: **Keck telescope, Hawaii, USA**



- Nobel Prize in Physics 2020 for the discovery of SMBH with mass of $\approx 4 \times 10^6 M_{\odot}$ at Galactic Centre, based on observations of stars orbiting Sgr A*, particularly star S2

Detection of gravitational redshift and Schwarzschild precession in the S2 star orbit

- In 2018, GRAVITY Collaboration detected the **gravitational redshift** in the orbit of the S2 star around the SMBH at the Galactic Centre
- Detected combined transverse Doppler and gravitational redshift up to $200 \text{ km/s}/c$ (right figure) was in agreement with GR predictions
- In 2020, GRAVITY Collaboration also detected the **Schwarzschild precession** in the S2 star orbit



Left: projected precession angle as a function of time (residuals between the best-fit GR orbit and the Newton orbit + Rømer effect + transverse Doppler effect + gravitational redshift)

Lense-Thirring precession around rotating SMBHs

- A star orbiting a spinning (Kerr) SMBH experiences Lense-Thirring (LT) precession, causing its orbital line of nodes to precess at a rate:

$$\frac{d\Omega}{dt} = \frac{2G^2 M^2 \chi}{c^3 a^3 (1 - e^2)^{\frac{3}{2}}},$$

where a and e are the semimajor axis and eccentricity of the orbit, M is the mass of the SMBH, and χ is the dimensionless spin parameter ($0 < \chi < 1$)

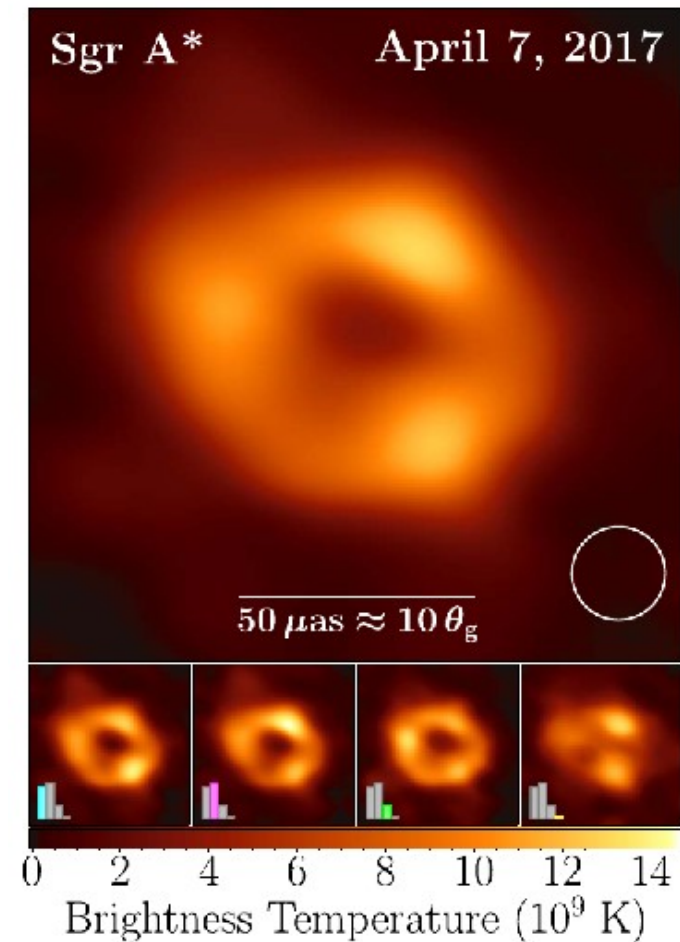
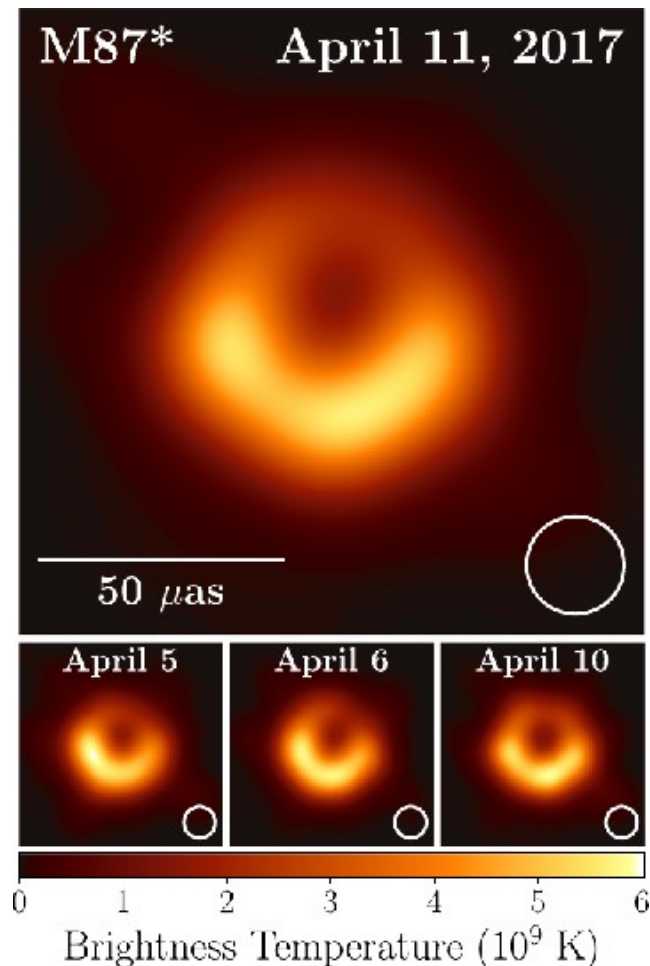
- It should be possible to detect the LT precession of S-stars by long-term monitoring of their orbits around Sgr A*
- The detected LT precession could be then used for constraining the spin χ of Sgr A*

Star	a (mpc)	P (yr)	e	$\dot{\Omega}_{\max}^{\text{LT}}$ (" yr ⁻¹)	$\dot{\Omega}_{\min}^{\text{LT}}$ (" yr ⁻¹)
S4714	4.079 ± 0.012	12.0 ± 0.3	0.985 ± 0.011	8.9	-8.9
S62	3.588 ± 0.02	9.9 ± 0.3	0.976 ± 0.01	5.15	-5.32
S4711	3.002 ± 0.06	7.6 ± 0.3	0.768 ± 0.030	0.35	-0.34

Maximum and minimum values of the LT rate of change of the node Ω of 3 selected short-period S-stars (Iorio, 2020, ApJ, 904, 186)

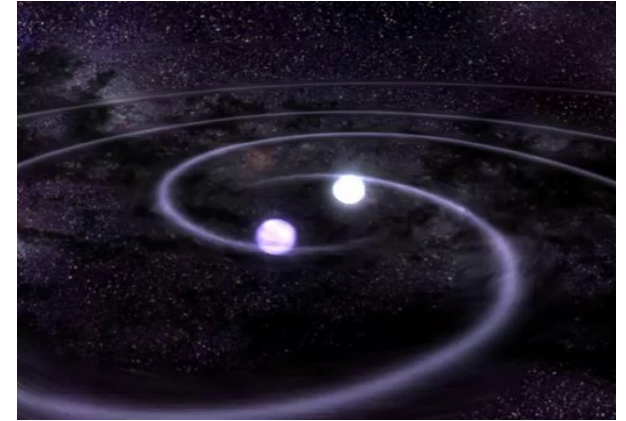
Observed images of the SMBH shadows at the centers of M87 and Milky Way

- When surrounded by a transparent emission region, black holes are expected to reveal a dark shadow caused by gravitational light bending and photon capture at the event horizon
- Such shadows were imaged in 2017 by the Event Horizon Telescope (EHT) in the case of central SMBHs of M87 galaxy and the Milky Way
- These images are consistent with expectations for the shadow of a rotating (Kerr) SMBH

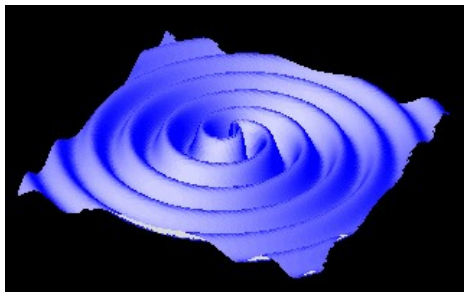


Gravitational waves and their polarizations

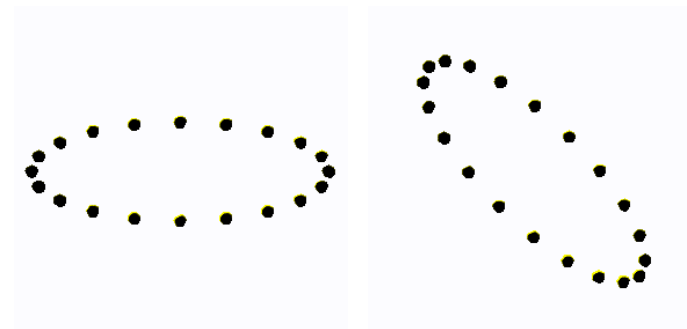
- **Gravitational waves (GW)** are ripples in spacetime caused by massive objects moving with extreme accelerations (such as neutron stars or black holes orbiting each other) and disrupting spacetime in such a way that waves of undulating spacetime would propagate in all directions away from the source



- In the frame of GR, Einstein predicted the existence of GW in 1916
- For GW emitted from a binary system at distance R with components of masses m_1 and m_2 :
- **GW amplitude** is: $h_0 = \frac{2\eta}{c^4 R} \frac{(Gm)^2}{p}$, where $\eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$ is symmetric mass ratio of the system, $p = a(1 - e^2)$ is the orbit's semilatus rectum and $m = m_1 + m_2$
- **GW polarizations** are: $h_+ = h_0 H_+$, $h_\times = h_0 H_\times$
- H_+ and H_\times are scale-free polarizations, which for a circular orbit are given by:
 $H_+ = -(1 + \cos^2 i) \cos 2(\Omega\tau + \omega)$, $H_\times = -2 \cos i \sin 2(\Omega\tau + \omega)$, where $\Omega = \sqrt{Gm/p^3}$ and $\tau = t - R/c$ is retarded time

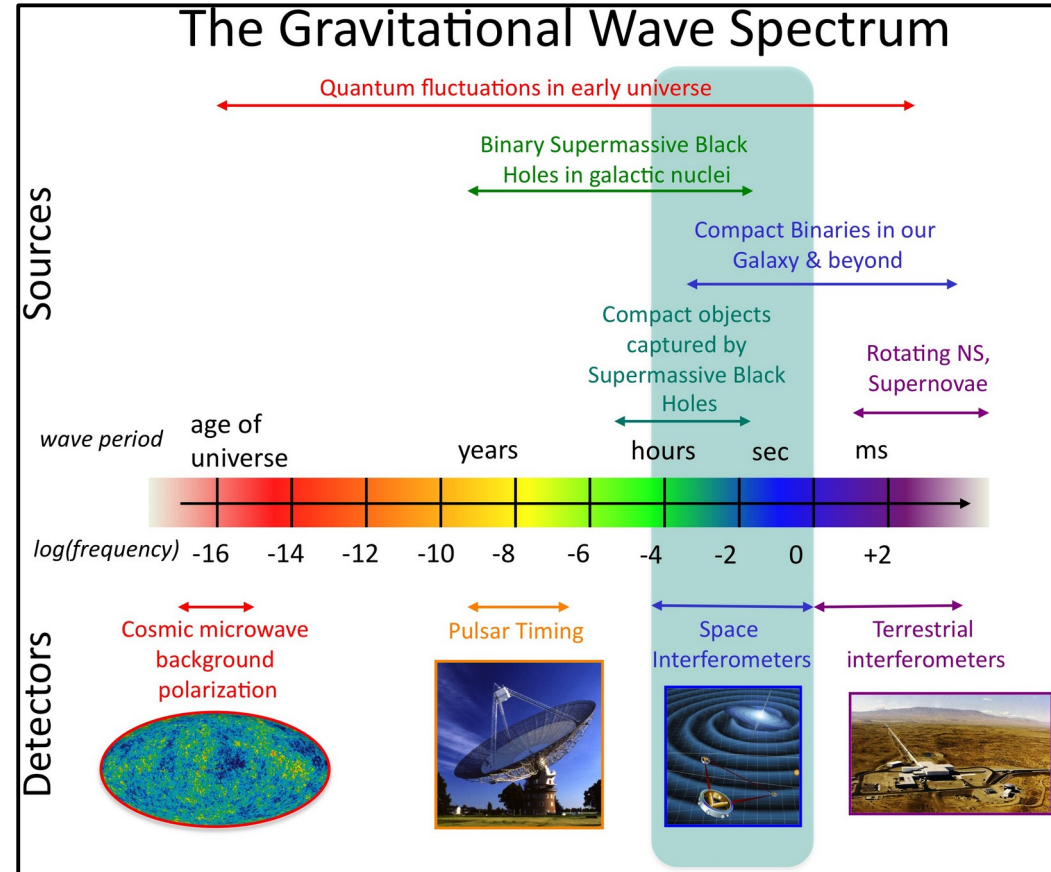
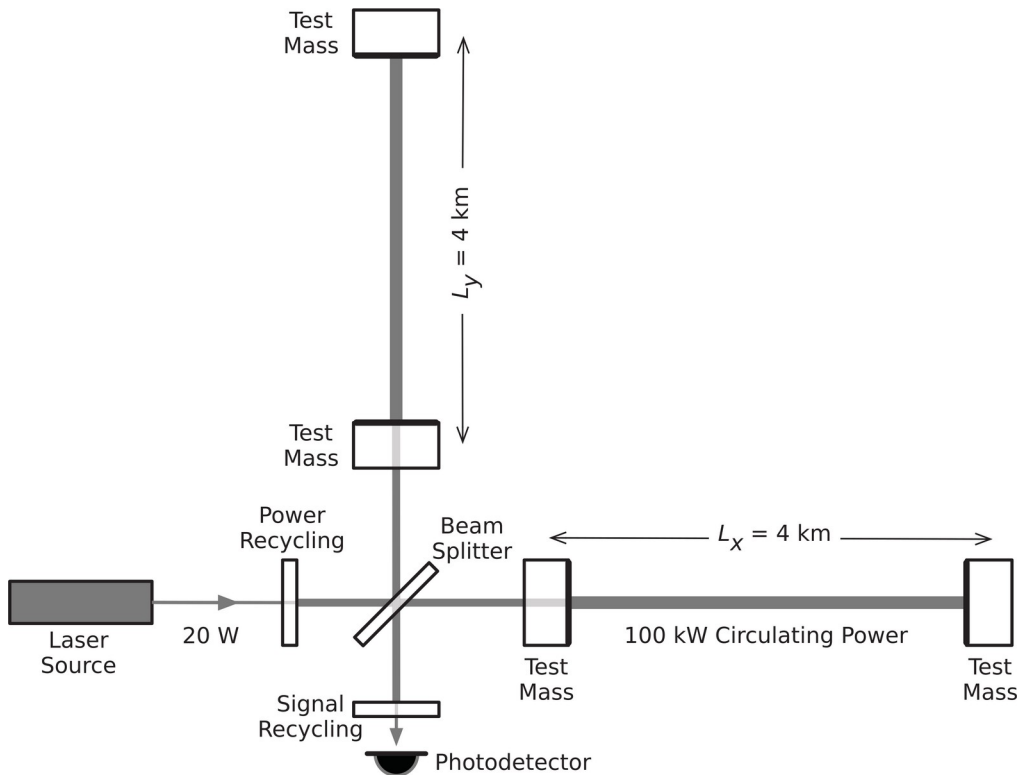


The effects of a plus-polarized (left) and cross-polarized (right) GW on a ring of particles



Astrophysical sources of GWs

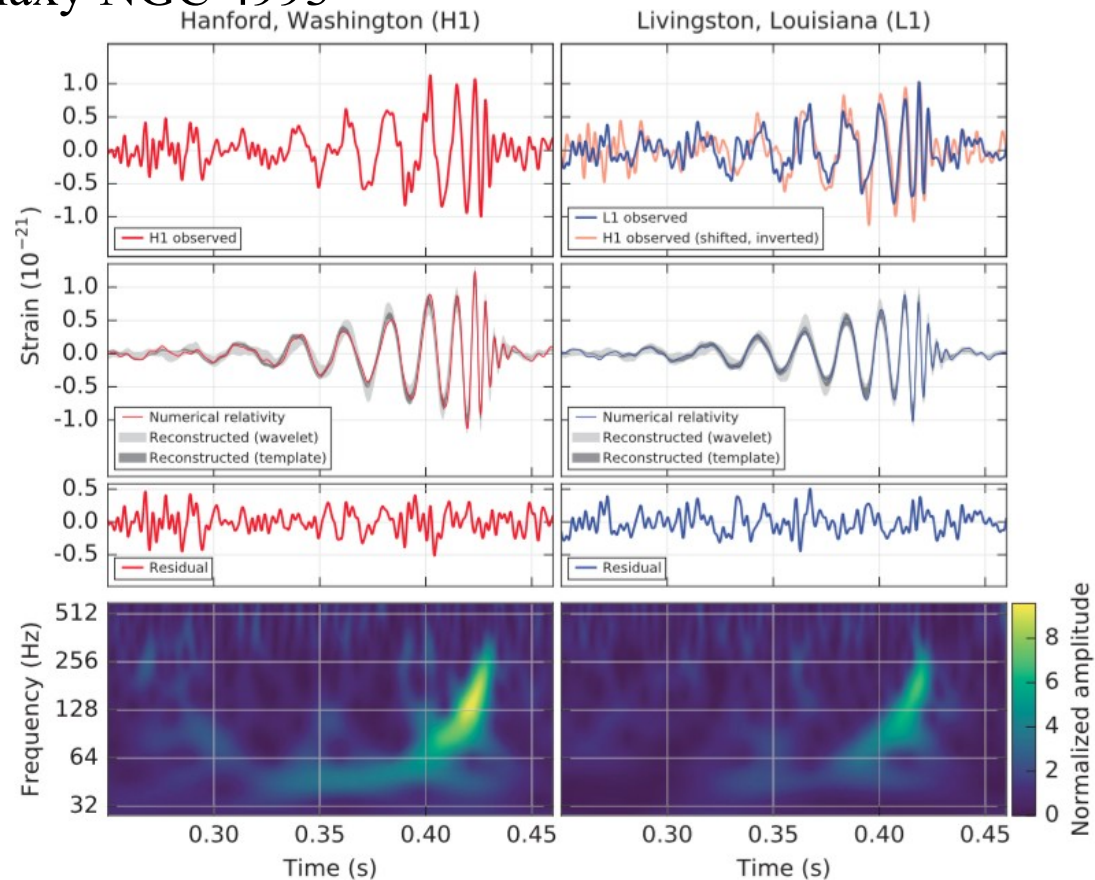
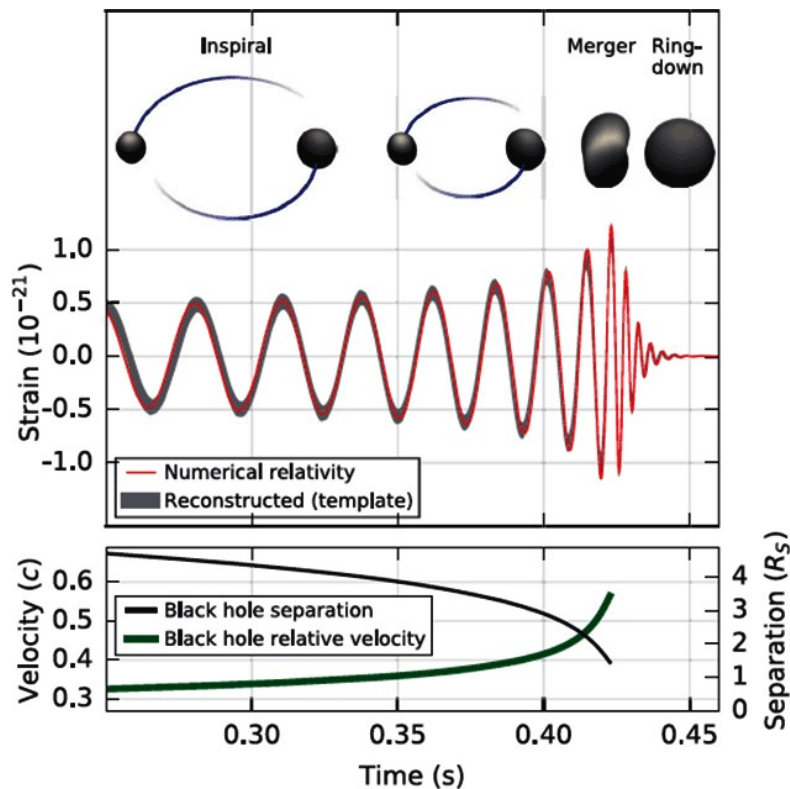
- Possible astrophysical sources of GW:
 - binary systems of white dwarfs, neutron stars and black holes
 - supernovae
 - cosmic inflation
- Depending on their frequency, GWs could be detected by ground-based (LIGO) or space-based (LISA) interferometers, as well as by pulsar timing arrays



- LIGO consists of two L-shaped detectors with 4 km long vacuum chambers
- It is capable of detecting a change in distance between its mirrors 1/10,000th the width of a proton, which is equivalent to measuring the distance to the nearest star (some 4.2 light years away) to an accuracy smaller than the width of a human hair

Detection of GWs

- On September 14, 2015, LIGO/Virgo Collaborations detected the first GW generated by two colliding black holes 1.3 billion light-years away
- Detected signal was in excellent agreement with the corresponding simulated waveform obtained by numerical relativity
- Nobel Prize in Physics 2017 was awarded to Rainer Weiss, Kip Thorne and Barry Barish for their role in the direct detection of GWs
- In 2017 LIGO/Virgo and Fermi/INTEGRAL made the joint detection of GW and γ -rays from a binary neutron star merger in the galaxy NGC 4993



Exam questions

1. Black holes, single and binary supermassive black holes (SMBHs) at the centers of Active Galactic Nuclei (AGN) and relativistically broadened spectral lines from AGN
2. Discovery of the central SMBH of the Milky Way and the observed images of the SMBH shadows at the centers of M87 and our Galaxy, gravitational waves and their detection

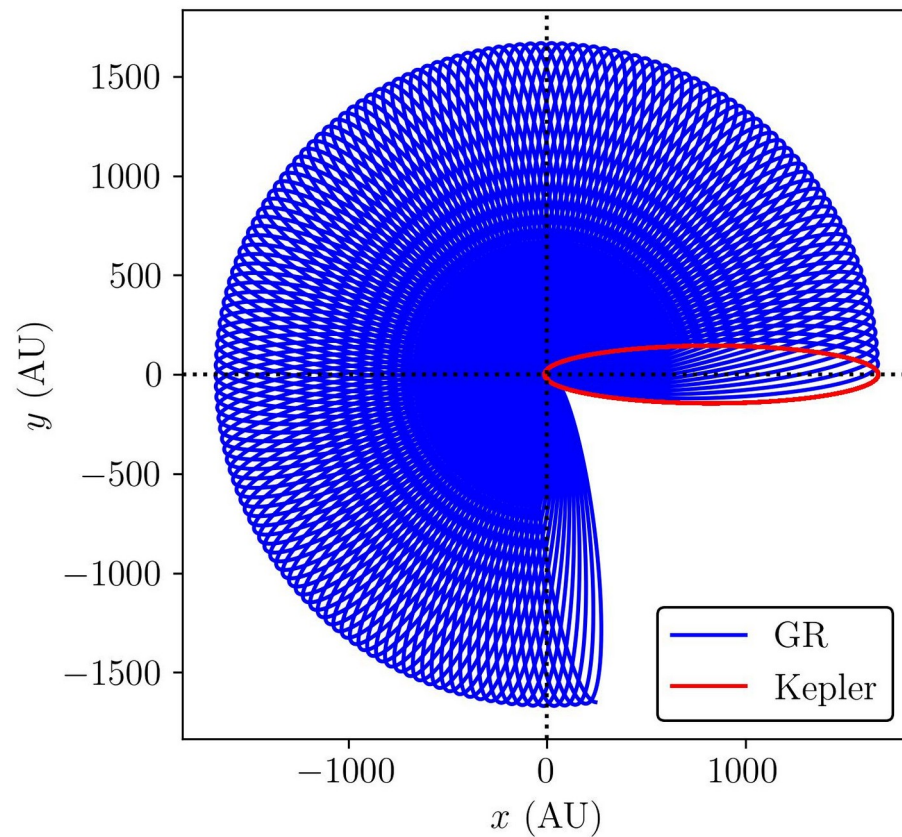
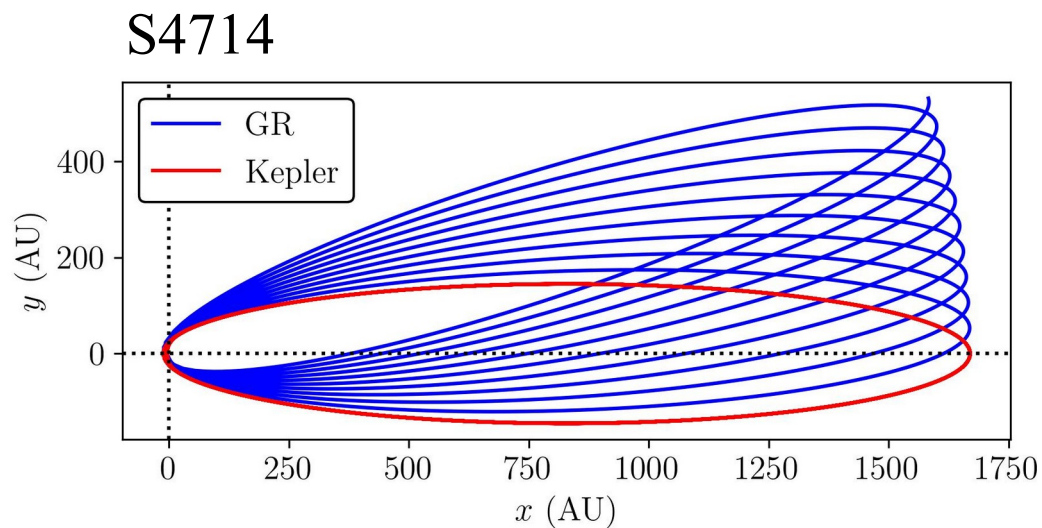
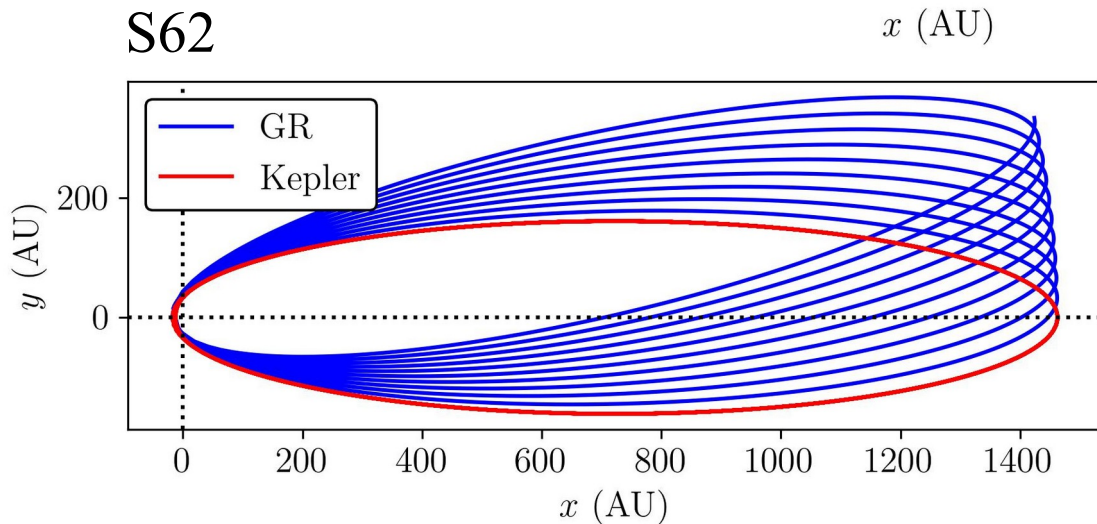
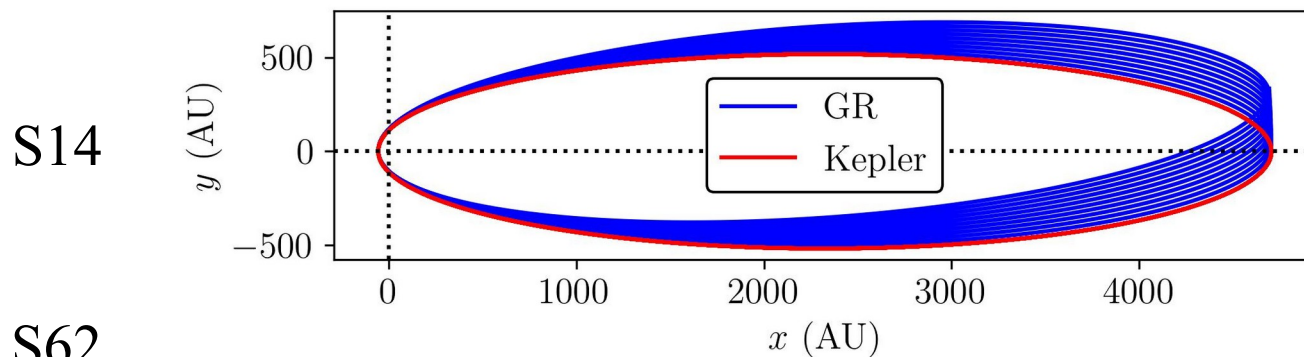
Literature (articles)

1. Event Horizon Telescope Collaboration, Akiyama, K., et al. 2022, ApJL, 930, L12
2. Event Horizon Telescope Collaboration, Akiyama, K., et al. 2019, ApJL, 875, L1
3. Everitt, C. W. F., et al. 2011, PhRvL, 106, 221101
4. GRAVITY Collaboration, Abuter, R., et al. 2020, A&A, 636, L5
5. GRAVITY Collaboration, Abuter, R., et al. 2018, A&A, 615, L15
6. Jovanović, P. 2012, NewAR, 56, 37
7. LIGO Scientific Collaboration and Virgo Collaboration, Fermi and INTEGRAL, Abbott, B. P., et al. 2017, ApJL, 848, L13
8. LIGO Scientific Collaboration and Virgo Collaboration, Abbott, B. P., et al. 2016, PhRvL, 116, 061102

Exercise 1

- Use the provided Python program "orbit.py" for two-body problem in General Relativity to simulate the orbits of the specified S-stars around Sgr A* (the central SMBH of the Milky Way) with mass of 4.3 million solar masses.
 - a) simulate the orbits of S14, S62 and S4714 star during 10 orbital periods, knowing that their semi-major axes a and eccentricities e are:
 - S14: $a = 2382$ AU, $e = 0.9761$
 - S62: $a = 740$ AU, $e = 0.976$ and
 - S4714: $a = 841$ AU, $e = 0.985$
 - b) simulate the orbit of S4714 star during 150 orbital periods and discuss the shapes of relativistic orbits

Solution 1



b) Two-body orbits in GR are rosette-shaped