MASS 2026 Course:
Gravitation and Cosmology

Predrag Jovanović
Astronomical Observatory Belgrade

Reminder to previous lecture

- **Comoving distance:**

$$D_C(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_M (1+z')^3 + \Omega_k (1+z')^2 + \Omega_\Lambda}}$$

- **Angular diameter distance:**

$$\Omega_k = 0 \Rightarrow D_A(z) = \frac{D_C(z)}{1+z}, \quad D_A(z_1, z_2) = \frac{D_C(z_2) - D_C(z_1)}{1+z_2}$$

- **Luminosity distance:**

$$\Omega_k = 0 \Rightarrow D_L(z) = (1+z) D_C(z) = (1+z)^2 D_A(z)$$

- **Comoving volume:**

$$\Omega_k = 0 \Rightarrow V_C(z) = \frac{4\pi}{3} D_C^3(z)$$

Lecture 11

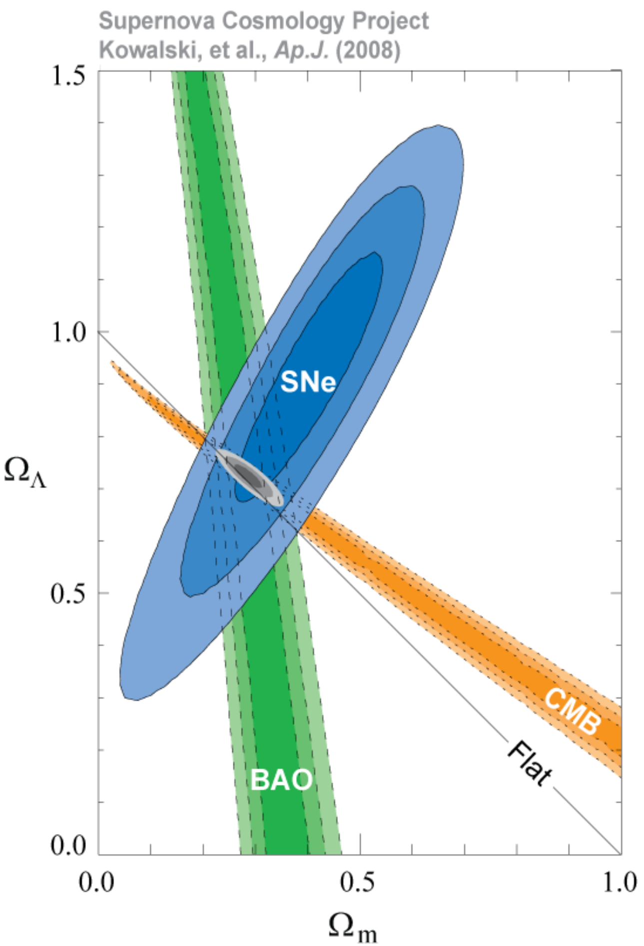
1) Observational cosmology
2) Cosmological tests:
   - **SN Ia**
   - **CMBR**
   - BAO
   - Faint galaxy counts
3) Exercises

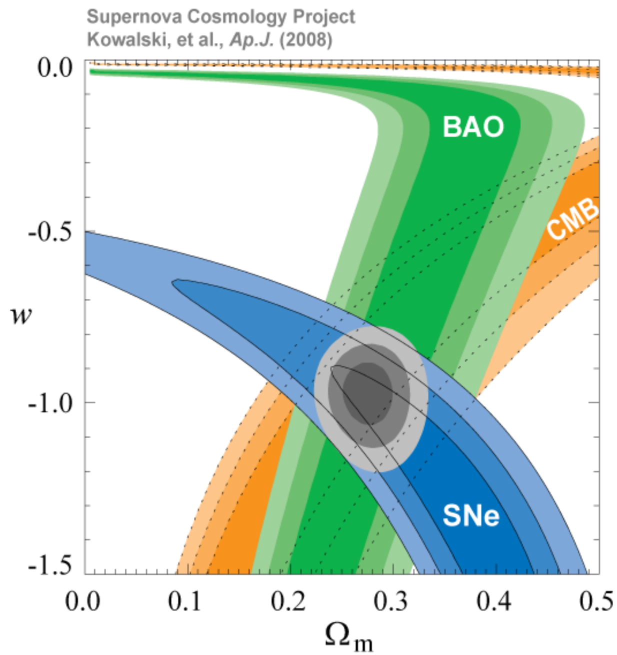
Observational cosmology

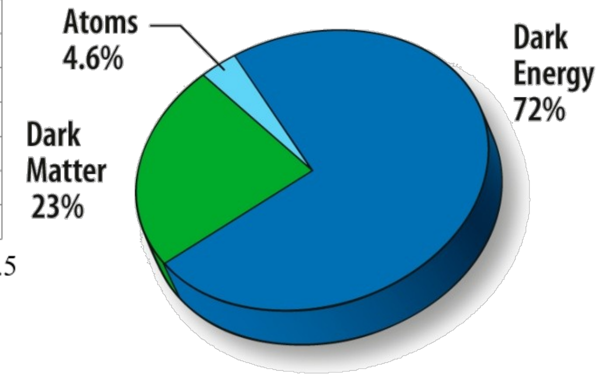
- **Goal:** study of origin, evolution and structure of the Universe using astronomical observations (cosmological tests)
- determination of cosmological parameters
- **Classification of cosmological tests:**

1. **using D_L:** SN Ia, Cepheids in other galaxies, Tully-Fisher and Faber-Jackson relation
2. **using D_A:** CMBR, BAO, GL
3. **using V_C, n or dP:** faint galaxy counts, GL statistics

Parameters of concordance cosmological model







- The most important result of standard ΛCDM cosmological model: more than 95% of present content of the Universe has unknown nature

Supernovae of type Ia

- Originate in close binary systems
- Standard candles because they have the same absolute magnitude *M* which is known (≈ -19^{m}.5)
- Apparent magnitude *m* is measured photometrically
- *z* is measured spectroscopically
- Relation between the magnitudes:

$$M = m - 5\log\frac{r}{\mathrm{pc}} + 5 \Leftrightarrow$$

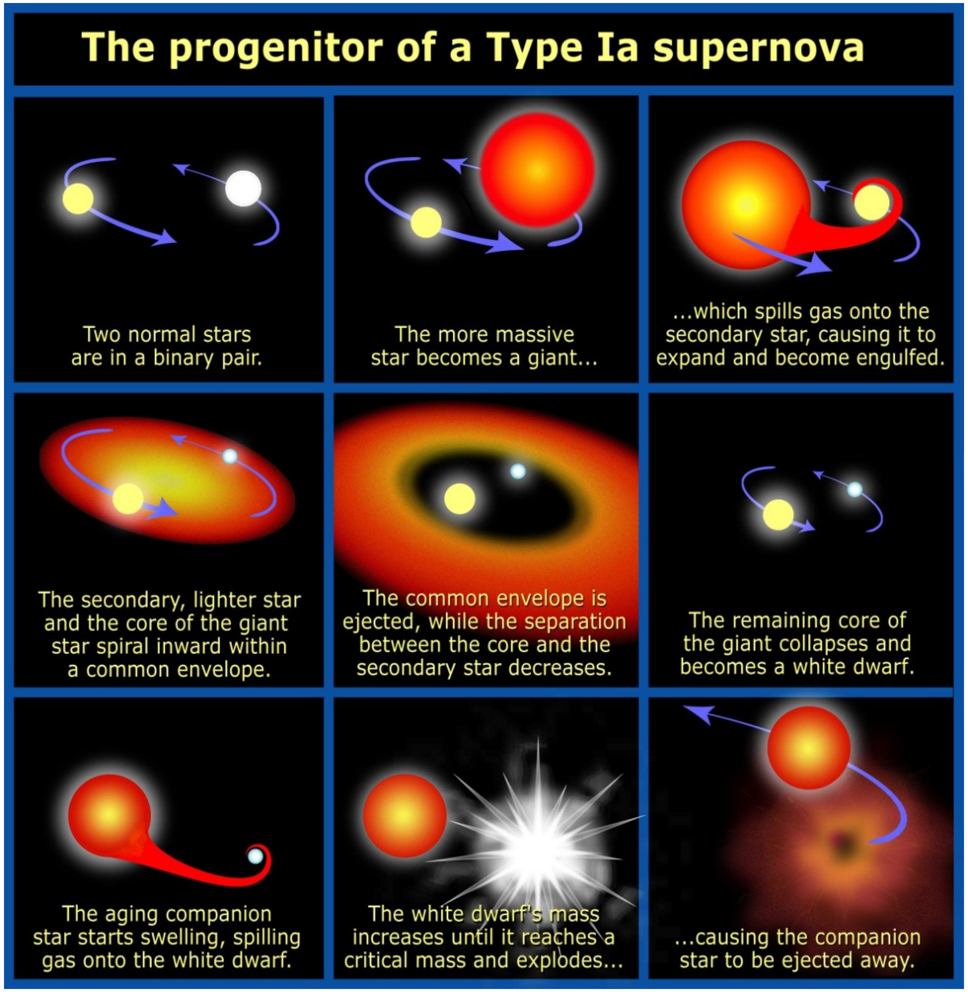
$$M = m - 5\log\frac{r}{\mathrm{Mpc}} - 25 \quad (*)$$

- **Distance modulus**: $\mu = m - M$

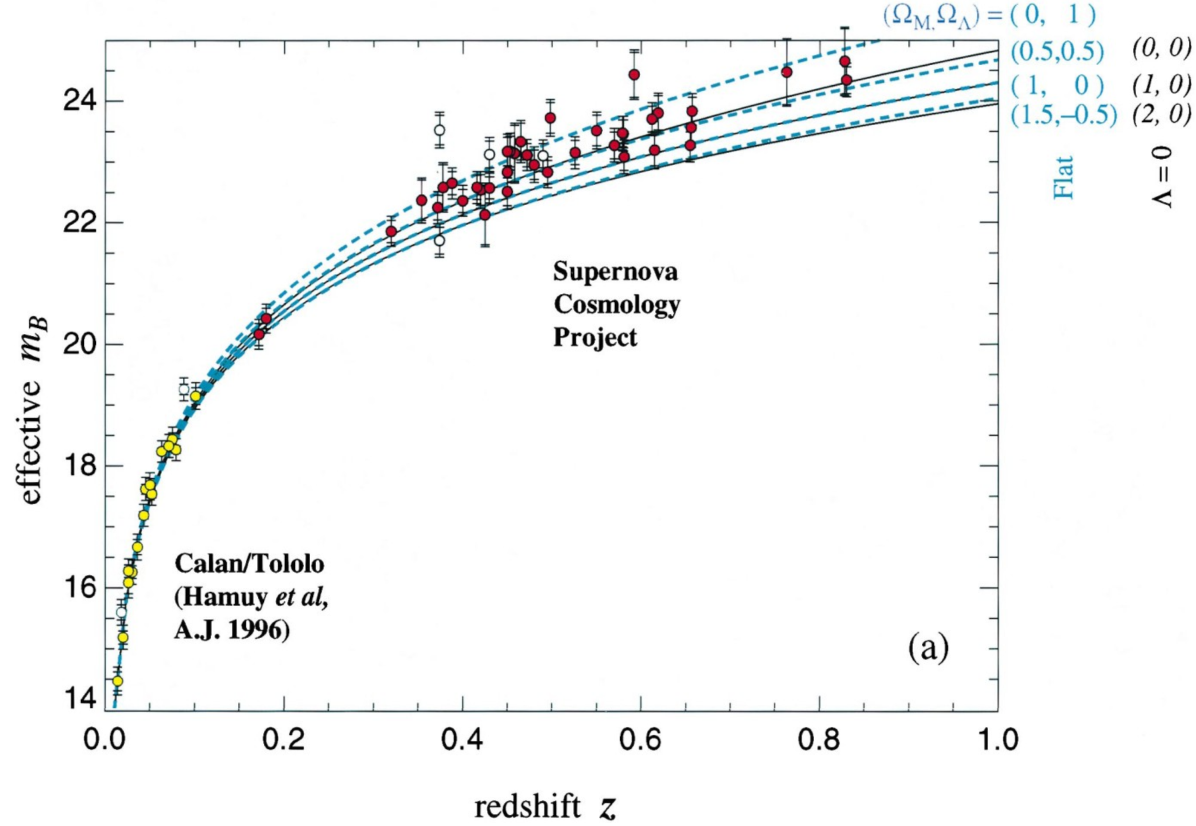
$$r\,[\mathrm{Mpc}] \longmapsto D_L\,[\mathrm{Mpc}] \overset{(*)}{\Rightarrow}$$

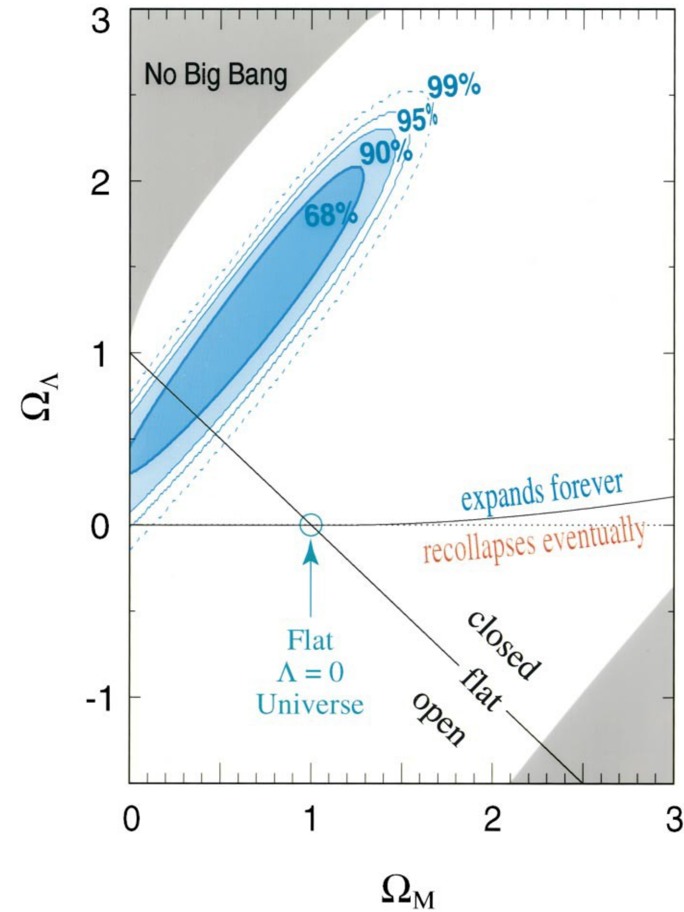
- Cosmological parameters are derived from (Perlmutter et al. ApJ, 1997, 483, 565):

$$\boxed{\mu(z) = 5\log D_L(z; H_0, \Omega_M, \Omega_\Lambda) + 25}$$

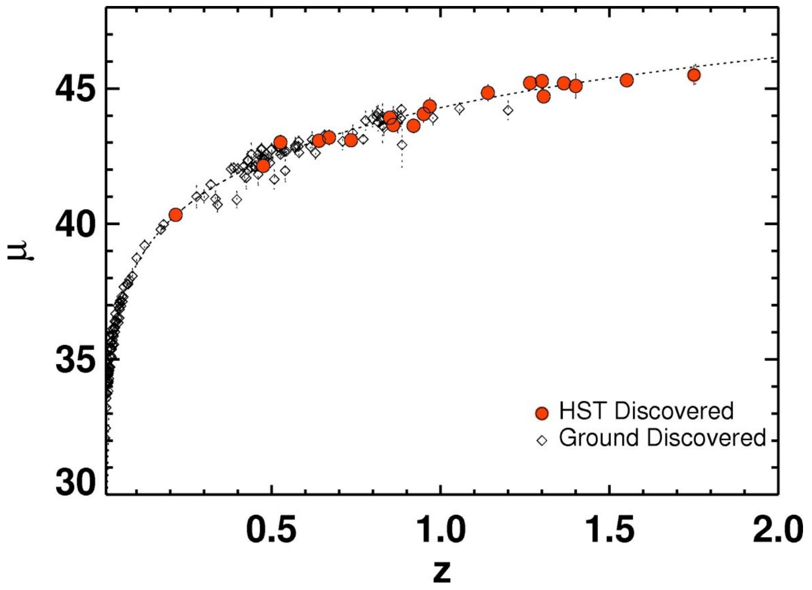


- "Supernova Cosmology Project"
- "High-Z SN Search"





(Perlmutter et al. 1999, ApJ, 517, 565):



(Riess et al. 2004, ApJ, 607, 665)

Finding q_0 from Hubble diagram of SN Ia

- Definitions of the Hubble, deceleration and jerk parameters:

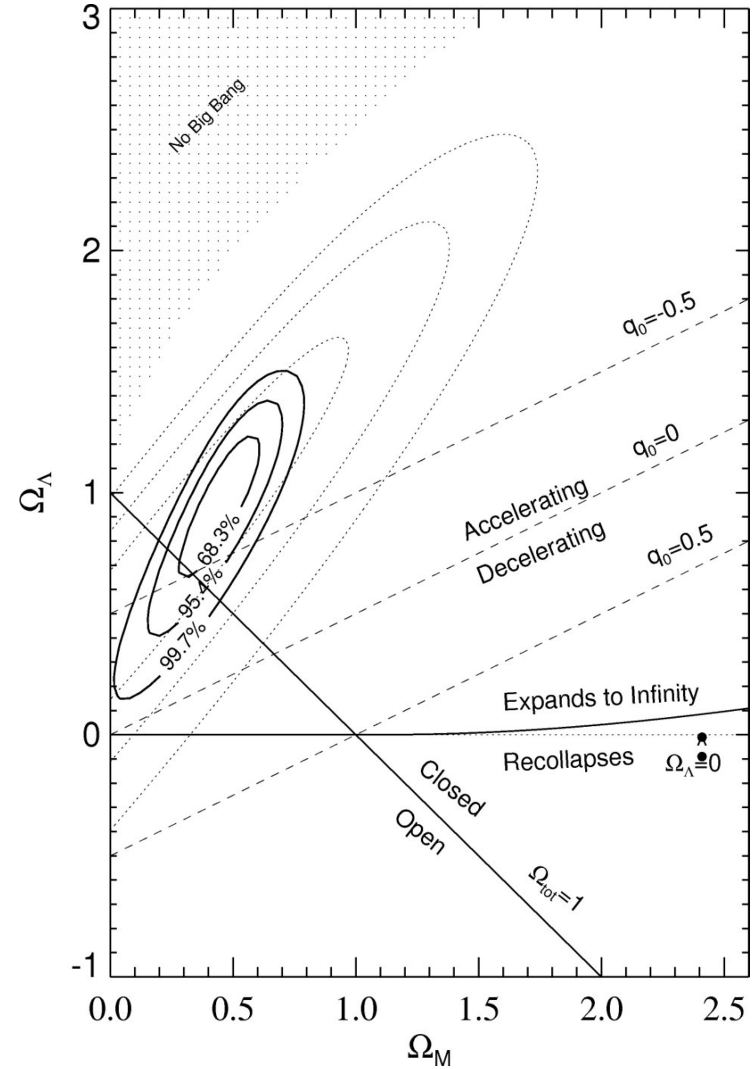
$$H(t) = +\dot{a}/a, \quad q(t) = -(\ddot{a}/a)(\dot{a}/a)^{-2},$$

$$j(t) = +(\dddot{a}/a)(\dot{a}/a)^{-3}$$

- Taylor series expansion of $a(t)$ around present epoch t_0:

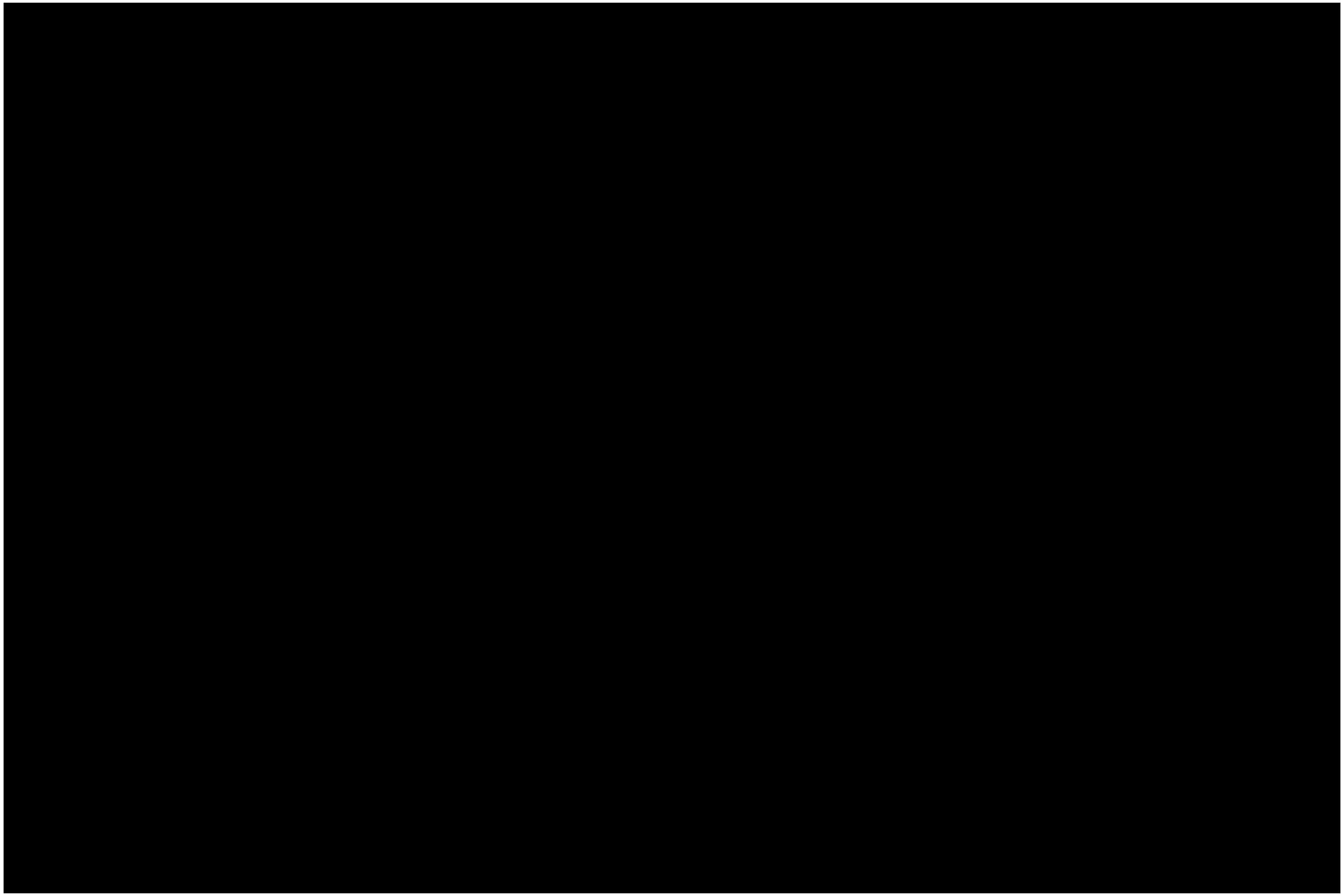
$$a(t) = a_0 \left\{ 1 + H_0(t-t_0) - \frac{1}{2} q_0 H_0^2 (t-t_0)^2 + \frac{1}{3!} j_0 H_0^3 (t-t_0)^3 + O\left[(t-t_0)^4\right] \right\}$$

- Relation between D_L and q_0 in Euclidean space (Riess et al. 2004, ApJ, 607, 665):



$$D_L = c(1+z)\int_0^z \frac{du}{H(u)} = \frac{cz}{H_0}\left[1 + \frac{1}{2}(1-q_0)z - \frac{1}{6}(1 - q_0 - 3q_0^2 + j_0)z^2 + O(z^3)\right]$$

Accelerating expansion of the Universe: Nobel Prize in physics 2011



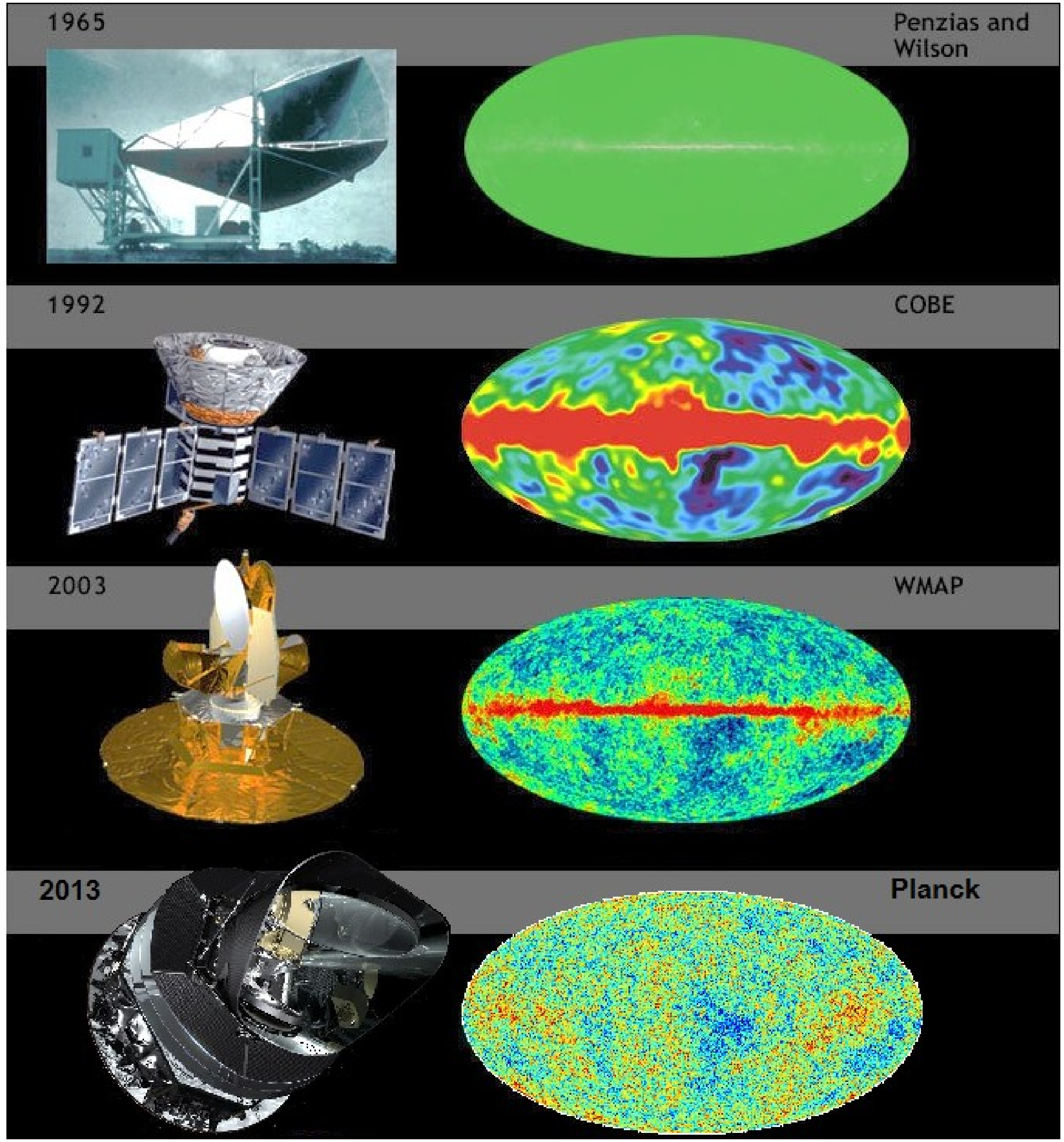
Saul Perlmutter

Brian P. Schmidt

Adam G. Riess

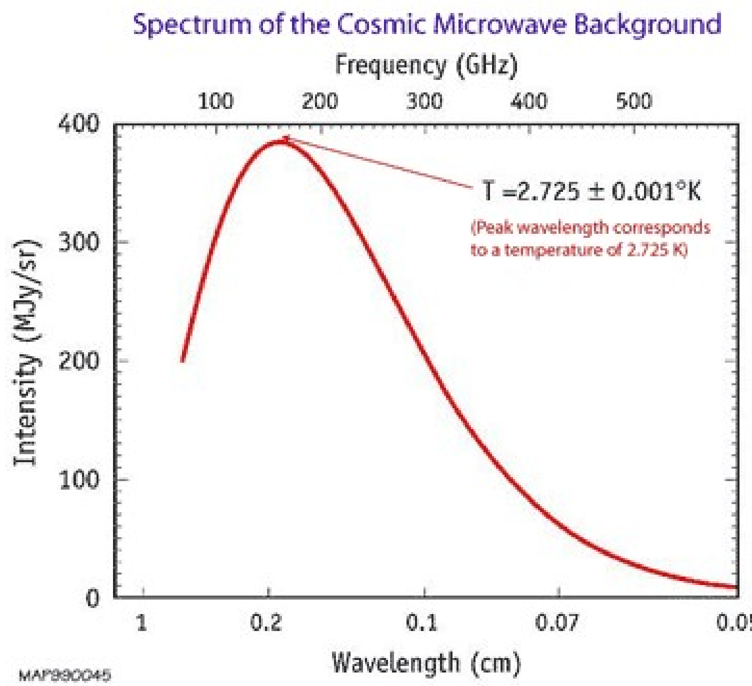
Cosmic Microwave Background Radiation (CMBR)

- Relict radiation from recombination epoch, about 380.000 years after Big Bang ($z \approx 1000$ - 1100)
- Due to cosmic expansion plasma cooled at about 3000 K which enabled recombination of protons and electrons into neutral hydrogen
- These atoms could no longer absorb the thermal radiation and photons starting to travel freely through the space (photon decoupling)
- CMBR temperature that we detect now is $T = 2.725$ K, which is about 1100 times less than in recombination epoch (because of increase of wavelength over time due to expansion of space)

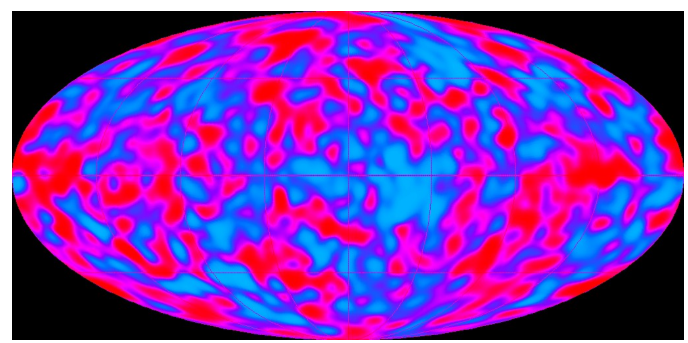


Cosmic Background Explorer (COBE)

- CMBR has Planck spectrum: the most perfect spectrum of black body radiation in nature
- Anisotropy (angular variations) in CMBR temperature due to slight variations in the density of the matter from which the light was last scattered
- These inhomogeneities were caused by quantum fluctuations in the inflaton field that caused the inflation
- Density perturbations in the early Universe act like acoustic waves due to opposite effects of gravitation and pressure: acoustic oscillations

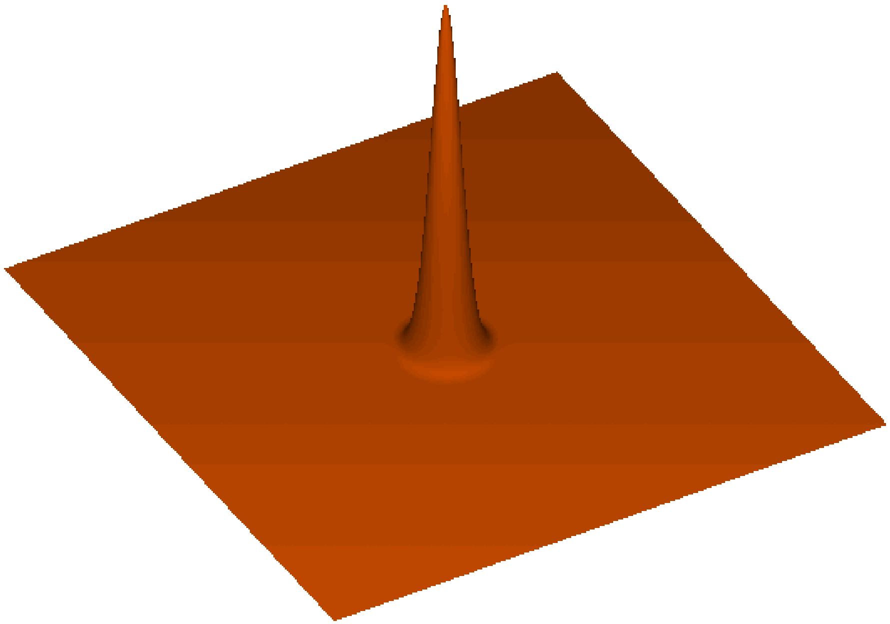


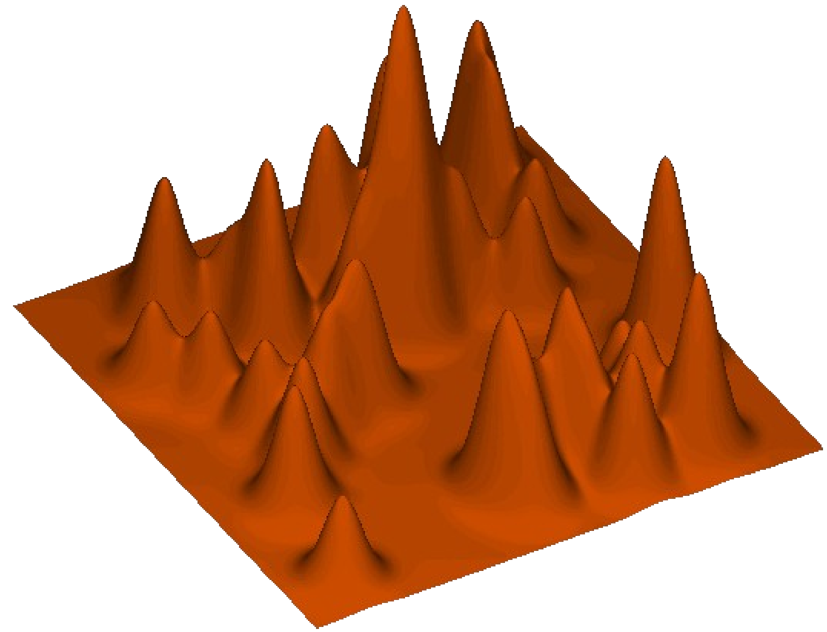
CMBR spectrum obtained by COBE



CMBR fluctuations (Galaxy removed)

Sound waves in the early Universe



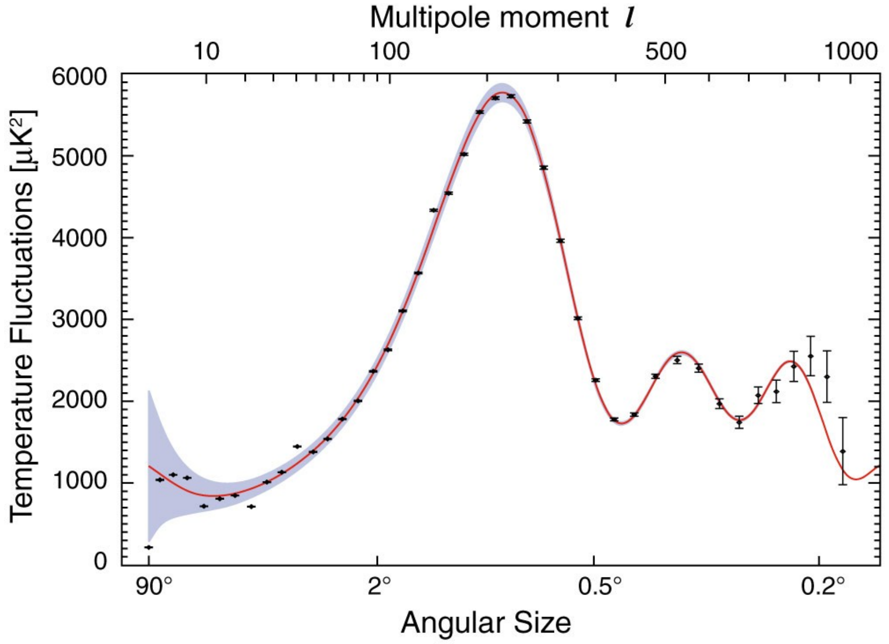


Left: a spherical sound wave from a small overdense region spreads until baryonic matter decouples from radiation. The radius of the acoustic shell (seen as a ring) represents a characteristic length scale: the sound speed times duration of this cosmological epoch

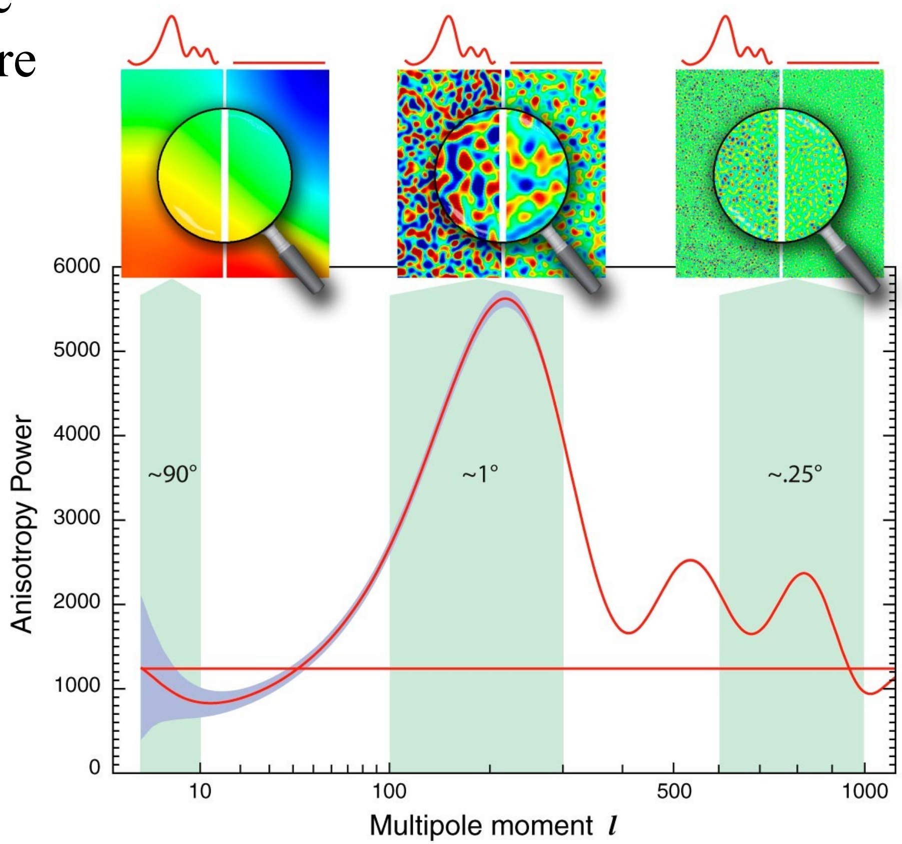
Right: superposition of spherical sound waves in the later universe. Characteristic size can still be extracted from statistical correlations in the large-scale distribution of galaxies.

Wilkinson Microwave Anisotropy Probe (WMAP)

- Order of magnitude of CMBR anisotropy: 10^{-5} K (cosmology as a precise observational science)
- **angular spectrum:** obtained by harmonic analysis of the angular sizes of temperature spots in CMBR (multipole expansion)



Angular spectrum: dependence of temperature fluctuation amplitudes on the angular sizes of the spots



CMBR temperature anisotropy for different angular sizes of the spots

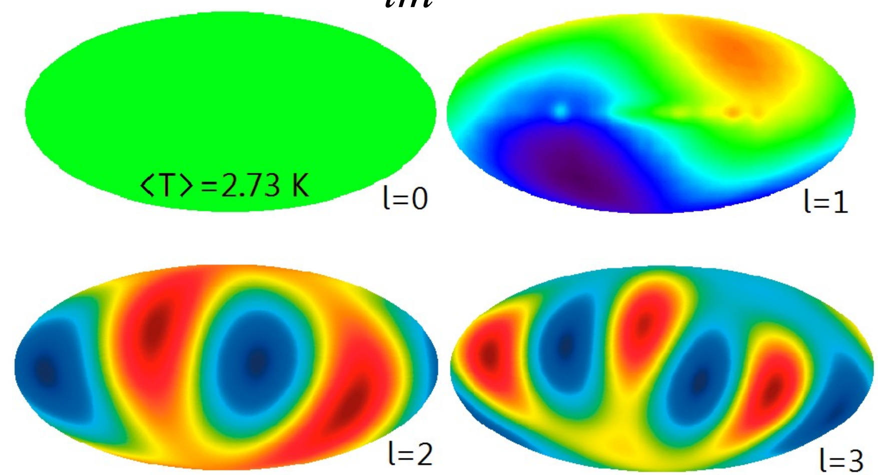
CMBR angular power spectrum

- CMBR anisotropy decomposition to the spherical harmonics $Y_{lm}(\theta, \phi)$:

$$\frac{T(\theta,\phi)-\langle T\rangle}{\langle T\rangle}=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}a_{lm}Y_{lm}(\theta,\phi)\,,\;Y_{lm}=\sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)e^{im\phi}$$

- P_l^m are the Legendre polynomials, harmonic index l is called **multipole**, representing an angular scale in the sky α: $\alpha = \pi / l$, a_{lm} are coefficients:

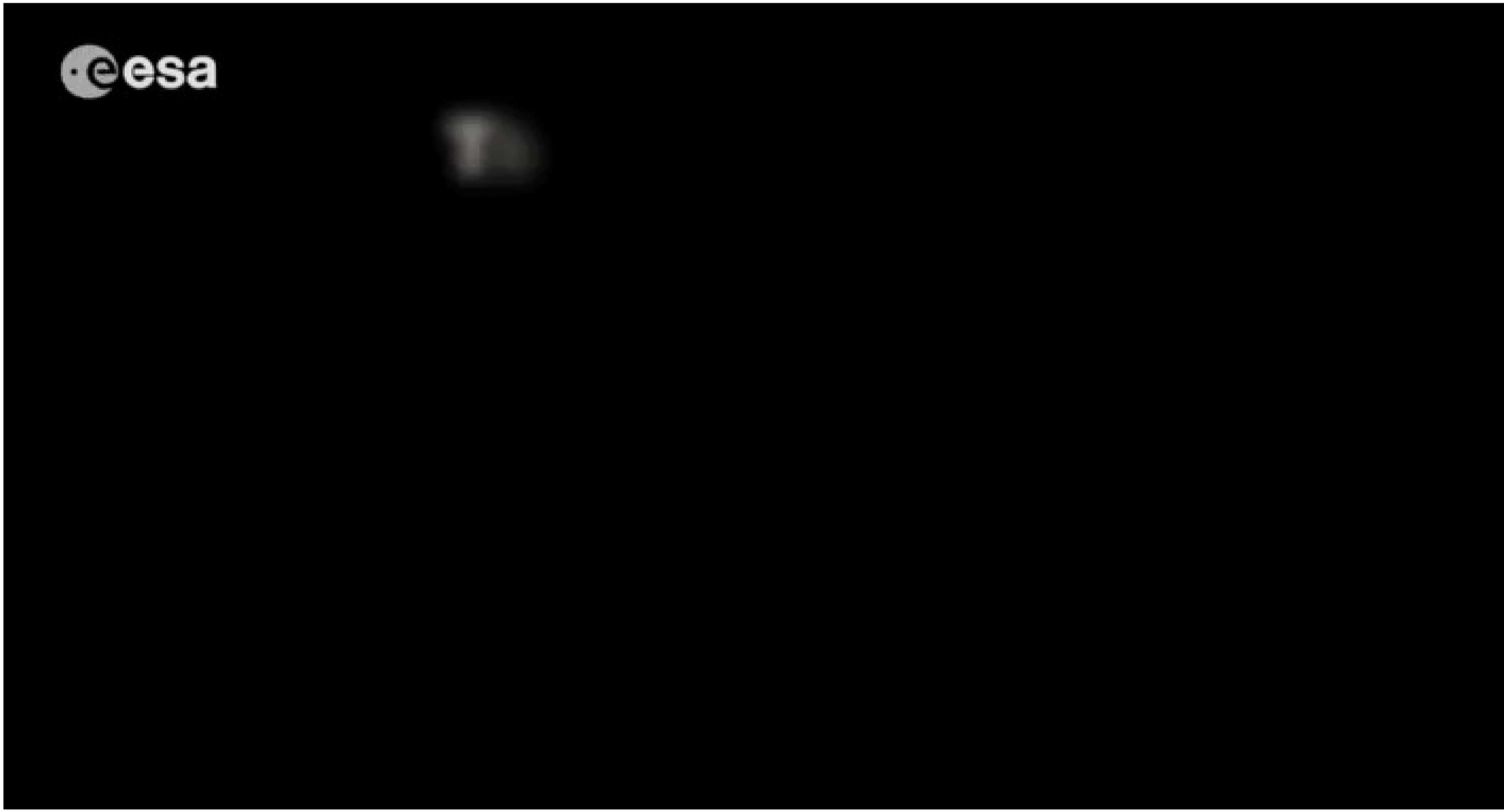
$$a_{lm}=\int_{\theta=-\pi}^{\pi}\int_{\phi=0}^{2\pi}\frac{T(\theta,\phi)-\langle T\rangle}{\langle T\rangle}Y_{lm}^{*}(\theta,\phi)\,d\Omega$$



- On small sections of the sky where its curvature can be neglected, the spherical harmonic analysis becomes ordinary 2D Fourier analysis
- **<u>Power spectrum (spectral density)</u>**:
- C_l is related to the expectation value of the correlation of the temperature between two points in the sky

$$C_l=\frac{1}{2l+1}\sum_{m=-l}^{l}\langle|a_{lm}|^2\rangle$$

CMBR angular power spectrum (Planck)

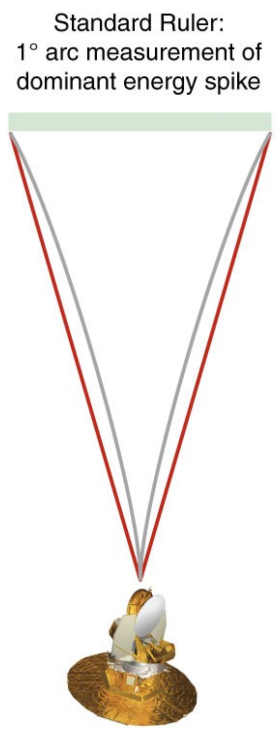


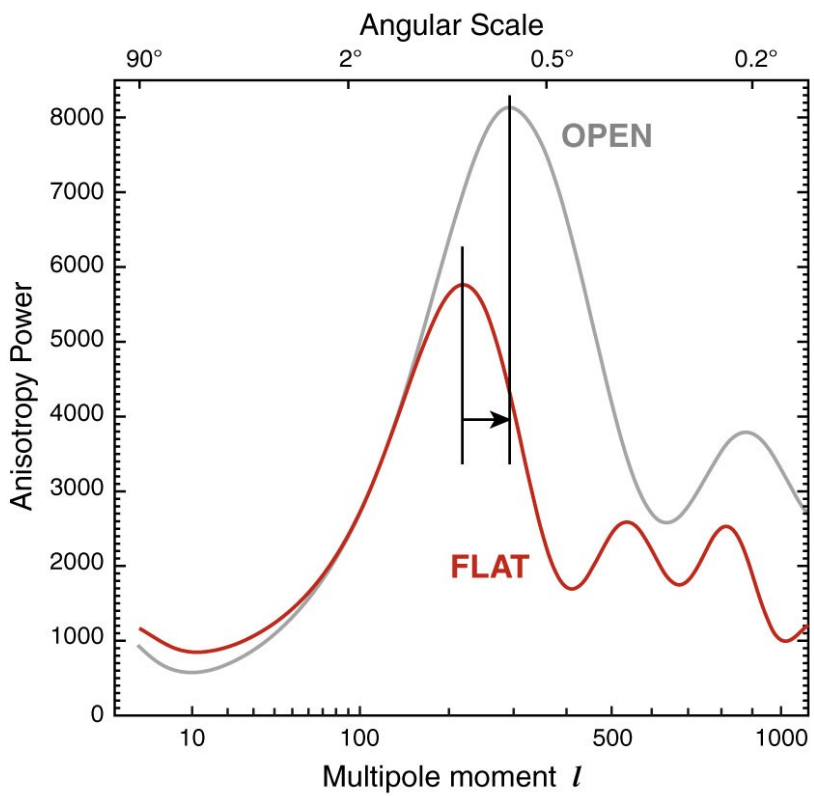
Determination of cosmological parameters from CMBR angular spectrum

- C_l contains all possible information about the underlying density fluctuations which are described by a Gaussian random process

- Cosmological parameters determined from angular scale θ of spots, i.e. from the peak positions l in angular spectrum:

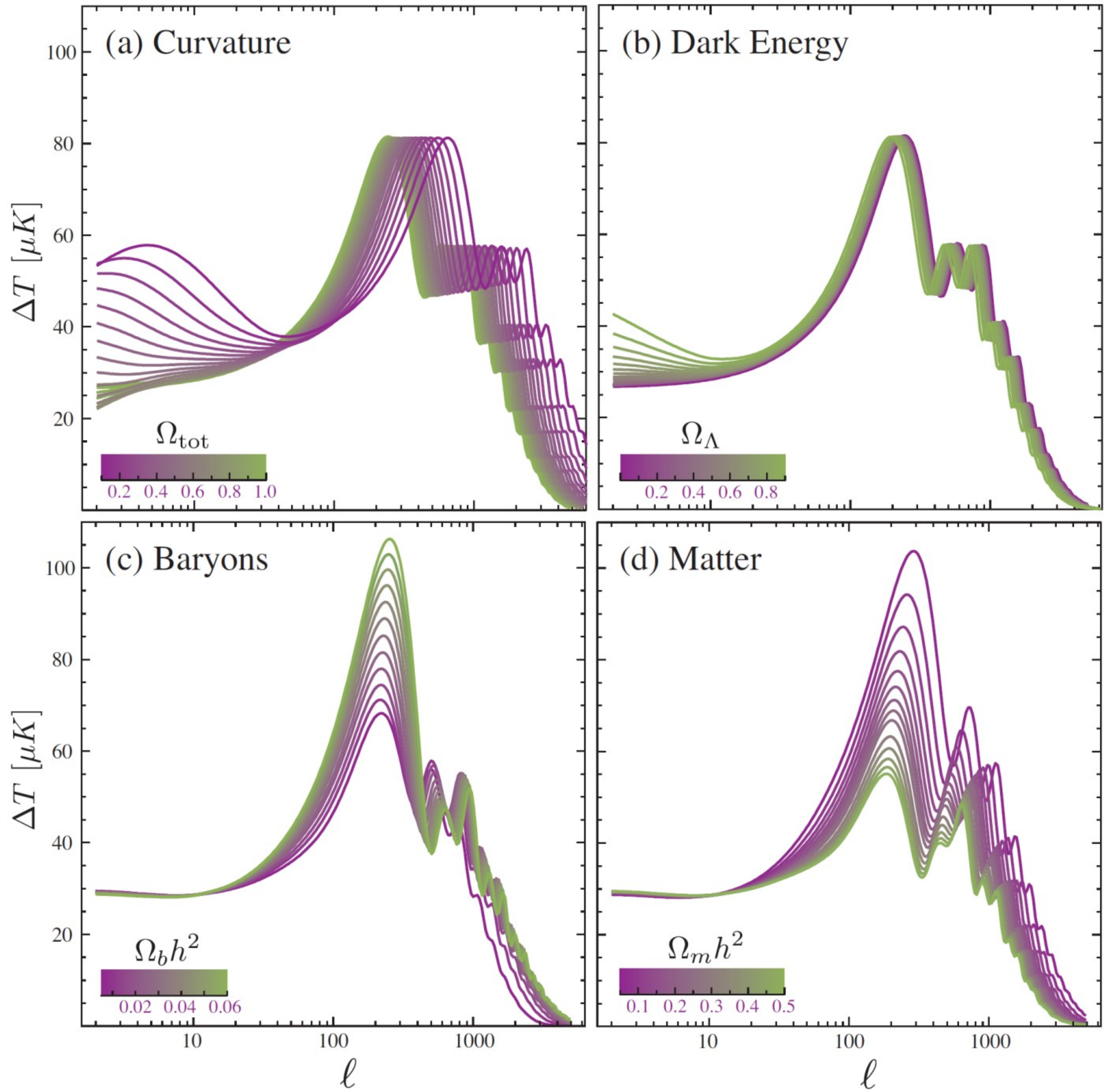
$$l = \frac{\pi}{\theta} = \frac{\pi D_A(z)}{r_s(z)}$$



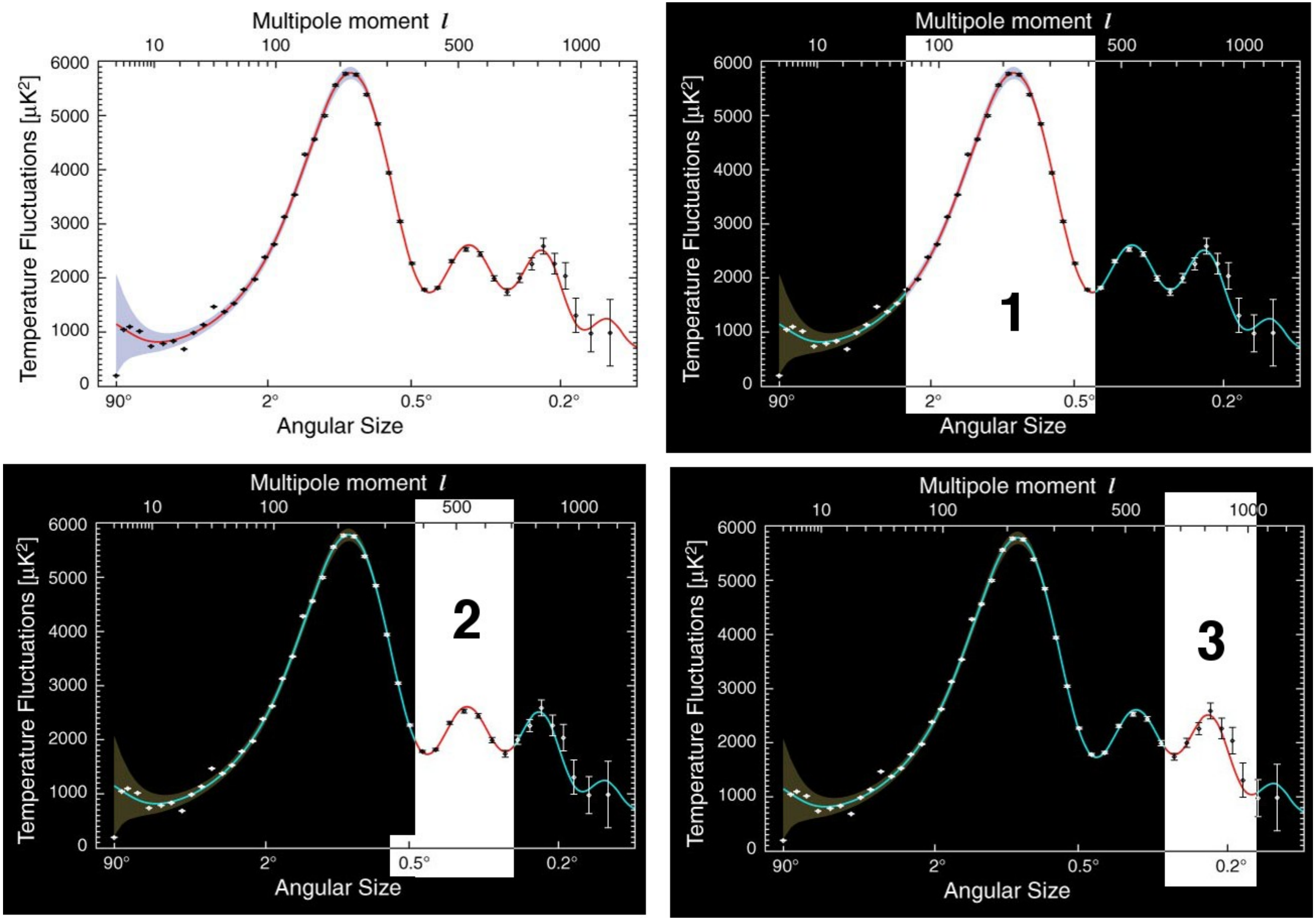


- θ - angular scale
- r_s - sound horizon radius: radius of density perturbation (sound wave) at which the photons decoupled from baryonic matter
- r_s (linear radii of spots) as standard rulers

Dependence of CMBR fluctuations on cosmological model



WMAP power spectrum



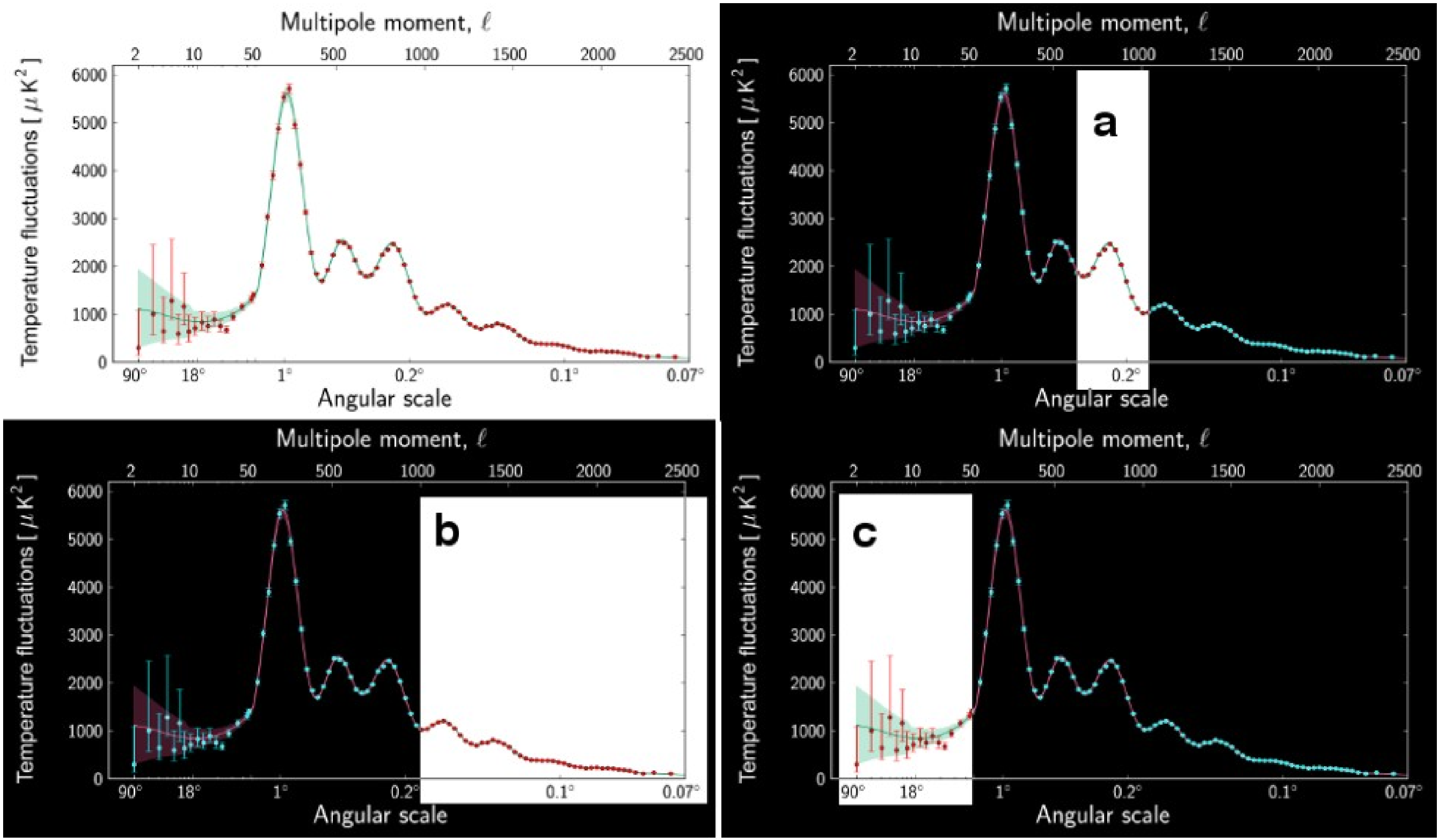
WMAP values of cosmological parameters

WMAP Seven-year Cosmological Parameter Summary

Description	Symbol	*WMAP*-only	*WMAP*+BAO+H_0
Parameters for the Standard ΛCDM Model[a]			
Age of universe	t_0	13.75 ± 0.13 Gyr	13.75 ± 0.11 Gyr
Hubble constant	H_0	71.0 ± 2.5 km s^{-1} Mpc^{-1}	$70.4^{+1.3}_{-1.4}$ km s^{-1} Mpc^{-1}
Baryon density	Ω_b	0.0449 ± 0.0028	0.0456 ± 0.0016
Physical baryon density	$\Omega_b h^2$	$0.02258^{+0.00057}_{-0.00056}$	0.02260 ± 0.00053
Dark matter density	Ω_c	0.222 ± 0.026	0.227 ± 0.014
Physical dark matter density	$\Omega_c h^2$	0.1109 ± 0.0056	0.1123 ± 0.0035
Dark energy density	Ω_Λ	0.734 ± 0.029	$0.728^{+0.015}_{-0.016}$
Curvature fluctuation amplitude, $k_0 = 0.002$ Mpc^{-1}[b]	$\Delta^2_{\mathcal{R}}$	$(2.43 \pm 0.11) \times 10^{-9}$	$(2.441^{+0.088}_{-0.092}) \times 10^{-9}$
Fluctuation amplitude at $8h^{-1}$ Mpc	σ_8	0.801 ± 0.030	0.809 ± 0.024
Scalar spectral index	n_s	0.963 ± 0.014	0.963 ± 0.012
Redshift of matter–radiation equality	$z_{\rm eq}$	3196^{+134}_{-133}	3232 ± 87
Angular diameter distance to matter–radiation eq.[c]	$d_A(z_{\rm eq})$	14281^{+158}_{-161} Mpc	14238^{+128}_{-129} Mpc
Redshift of decoupling	z_*	$1090.79^{+0.94}_{-0.92}$	$1090.89^{+0.68}_{-0.69}$
Age at decoupling	t_*	379164^{+5187}_{-5243} yr	377730^{+3205}_{-3200} yr
Angular diameter distance to decoupling[c,d]	$d_A(z_*)$	14116^{+160}_{-163} Mpc	14073^{+129}_{-130} Mpc
Sound horizon at decoupling[d]	$r_s(z_*)$	$146.6^{+1.5}_{-1.6}$ Mpc	146.2 ± 1.1 Mpc
Acoustic scale at decoupling[d]	$l_A(z_*)$	302.44 ± 0.80	302.40 ± 0.73
Reionization optical depth	τ	0.088 ± 0.015	0.087 ± 0.014
Redshift of reionization	$z_{\rm reion}$	10.5 ± 1.2	10.4 ± 1.2

(Jarosik et al. 2011, ApJS, 192, 14)

Planck power spectrum



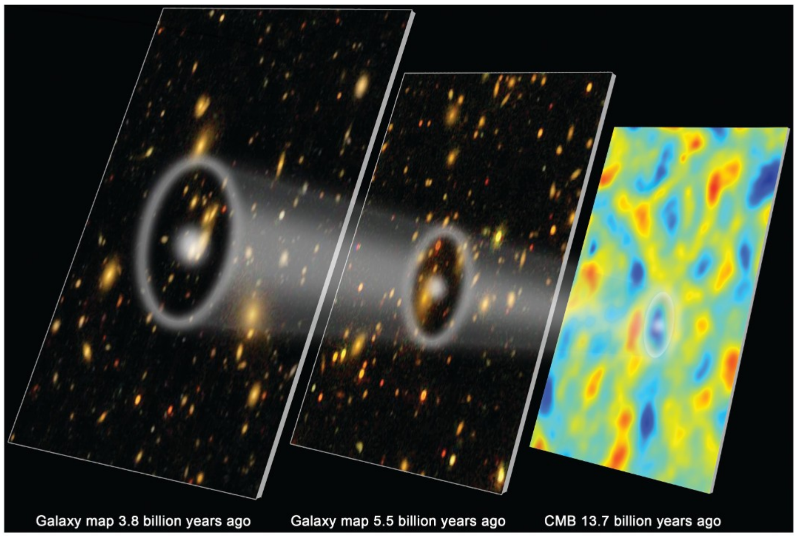
Planck values of cosmological parameters

Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022	0.02249 ± 0.00025	0.0240 ± 0.0012	0.02236 ± 0.00015	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1206 ± 0.0021	0.1177 ± 0.0020	0.1158 ± 0.0046	0.1202 ± 0.0014	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04077 ± 0.00047	1.04139 ± 0.00049	1.03999 ± 0.00089	1.04090 ± 0.00031	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0522 ± 0.0080	0.0496 ± 0.0085	0.0527 ± 0.0090	$0.0544^{+0.0070}_{-0.0081}$	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.040 ± 0.016	$3.018^{+0.020}_{-0.018}$	3.052 ± 0.022	3.045 ± 0.016	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9626 ± 0.0057	0.967 ± 0.011	0.980 ± 0.015	0.9649 ± 0.0044	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0 [km s^{-1} Mpc^{-1}] . .	66.88 ± 0.92	68.44 ± 0.91	69.9 ± 2.7	67.27 ± 0.60	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.679 ± 0.013	0.699 ± 0.012	$0.711^{+0.033}_{-0.026}$	0.6834 ± 0.0084	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.321 ± 0.013	0.301 ± 0.012	$0.289^{+0.026}_{-0.033}$	0.3166 ± 0.0084	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1434 ± 0.0020	0.1408 ± 0.0019	$0.1404^{+0.0034}_{-0.0039}$	0.1432 ± 0.0013	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09589 ± 0.00046	0.09635 ± 0.00051	$0.0981^{+0.0016}_{-0.0018}$	0.09633 ± 0.00029	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8118 ± 0.0089	0.793 ± 0.011	0.796 ± 0.018	0.8120 ± 0.0073	0.8111 ± 0.0060	0.8102 ± 0.0060
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$.	0.840 ± 0.024	0.794 ± 0.024	$0.781^{+0.052}_{-0.060}$	0.834 ± 0.016	0.832 ± 0.013	0.825 ± 0.011
$\sigma_8 \Omega_m^{0.25}$	0.611 ± 0.012	0.587 ± 0.012	0.583 ± 0.027	0.6090 ± 0.0081	0.6078 ± 0.0064	0.6051 ± 0.0058
z_{re}	7.50 ± 0.82	$7.11^{+0.91}_{-0.75}$	$7.10^{+0.87}_{-0.73}$	7.68 ± 0.79	7.67 ± 0.73	7.82 ± 0.71
$10^9 A_s$	2.092 ± 0.034	2.045 ± 0.041	2.116 ± 0.047	$2.101^{+0.031}_{-0.034}$	2.100 ± 0.030	2.105 ± 0.030
$10^9 A_s e^{-2\tau}$	1.884 ± 0.014	1.851 ± 0.018	1.904 ± 0.024	1.884 ± 0.012	1.883 ± 0.011	1.881 ± 0.010
Age [Gyr]	13.830 ± 0.037	13.761 ± 0.038	$13.64^{+0.16}_{-0.14}$	13.800 ± 0.024	13.797 ± 0.023	13.787 ± 0.020
z_*	1090.30 ± 0.41	1089.57 ± 0.42	$1087.8^{+1.6}_{-1.7}$	1089.95 ± 0.27	1089.92 ± 0.25	1089.80 ± 0.21
r_* [Mpc]	144.46 ± 0.48	144.95 ± 0.48	144.29 ± 0.64	144.39 ± 0.30	144.43 ± 0.26	144.57 ± 0.22
$100\theta_*$	1.04097 ± 0.00046	1.04156 ± 0.00049	1.04001 ± 0.00086	1.04109 ± 0.00030	1.04110 ± 0.00031	1.04119 ± 0.00029

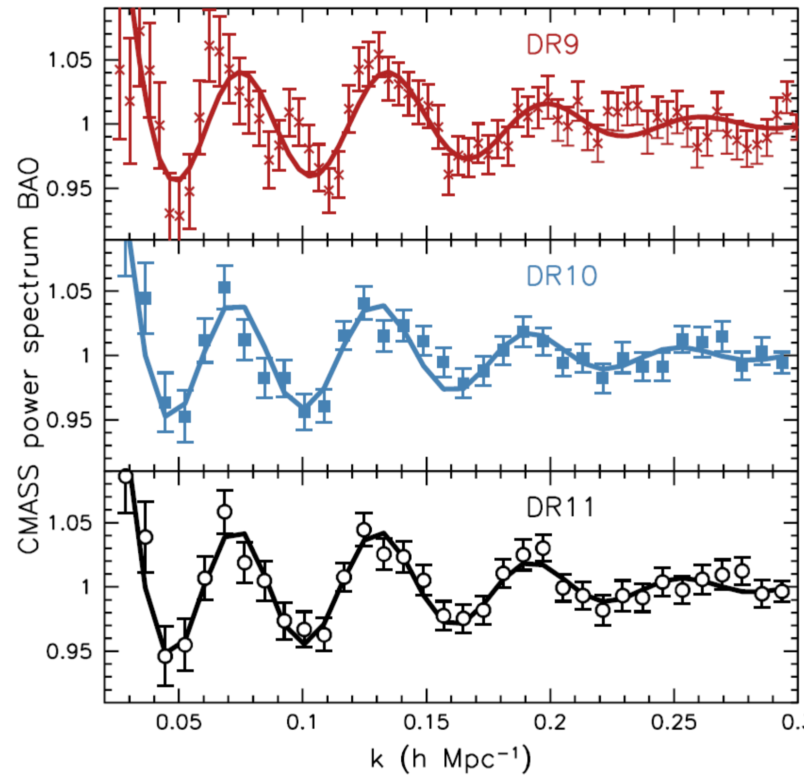
(Planck Collaboration, Aghanim et al. 2020. A&A, 641, A6)

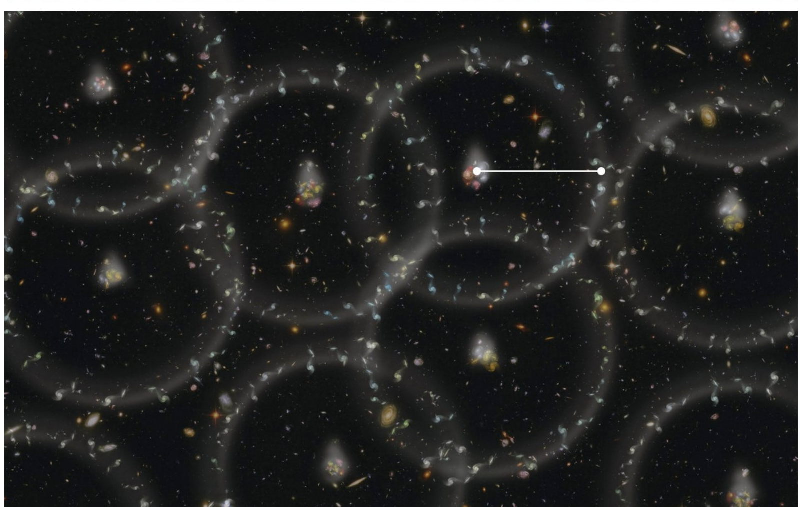
Baryon acoustic oscillations (BAO)

- Imprints of the sound waves from the era of CMBR onto the distribution of galaxies that we see today: galaxies have a tendency to align on the spheres
- Characteristic distance between galaxies provides a standard ruler



Anderson et al. 2014, MNRAS, 441, 24

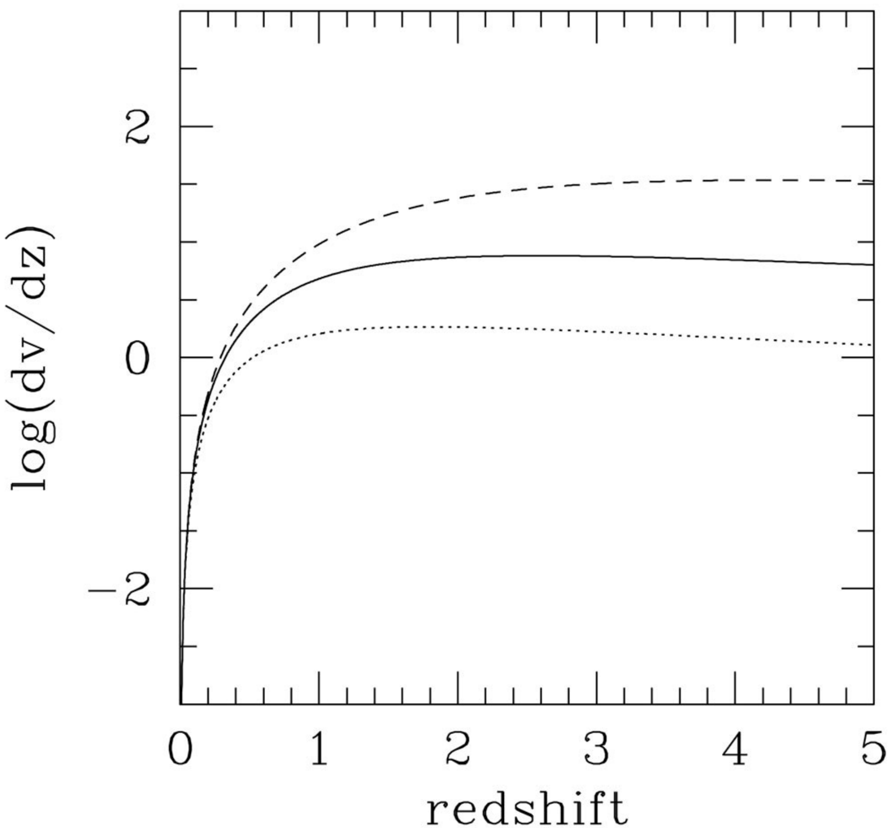


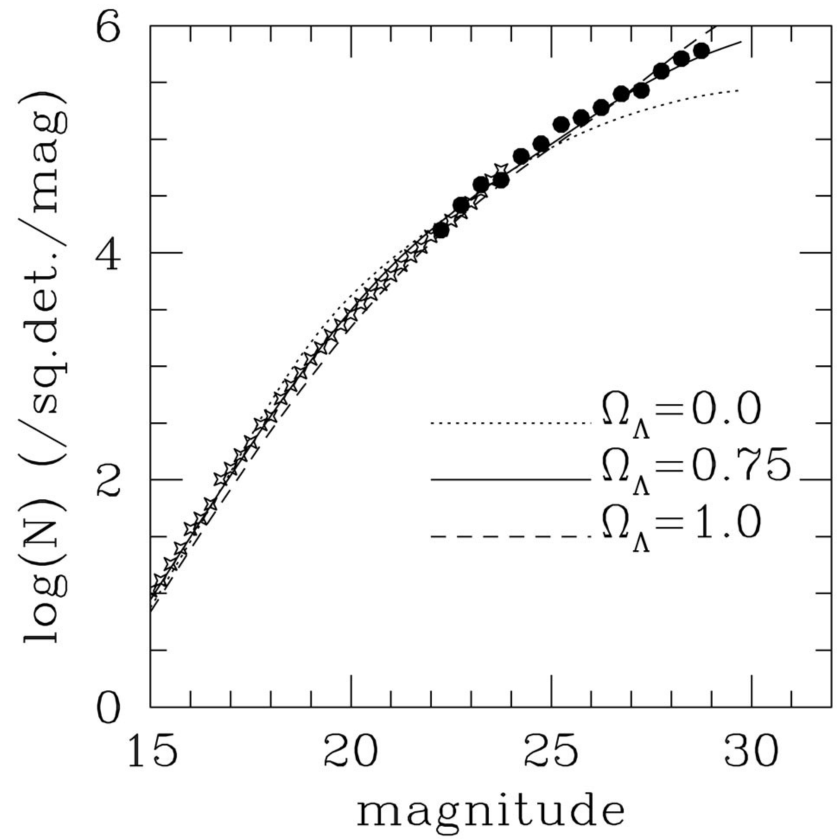


$$\theta = \frac{\pi}{l} = \frac{\pi}{k(1+z)D_A(z)}$$

- θ - angular scale
- l - multipole
- k - wavenumber

Number counts of faint galaxies





Left: incremental volume (in units of Hubble volume) per incremental redshift as a function of redshift for three flat cosmological models

Right: comparison between the observed faint galaxy number counts (ground based - open stars; Hubble Deep Fields - solid circles) with the predictions of three flat cosmological models

Exam questions

1. SN Ia as cosmological test
2. CMBR as cosmological test

Literature

- **Textbook:**
  1. Weinberg, S. 2008, *Cosmology*, Oxford University Press Inc., New York, USA
- **Articles:**
  1. Planck Collaboration, 2020, *Planck 2018 results. V. CMB power spectra and likelihoods*, A&A, 641, A5
  2. Planck Collaboration 2020, *Planck 2018 results. VI. Cosmological parameters*, A&A, 641, A6
  3. Perlmutter, S. et al. 1997, *Measurements of the Cosmological Parameters Ω and Λ from the First Seven Supernovae at $z \geq 0.35$*, ApJ, 483, 565
  4. Perlmutter, S. et al. 1999, *Measurements of Ω and Λ from 42 High-Redshift Supernovae*, ApJ, 517, 565
  5. Riess et al. 2004, *Type Ia Supernova Discoveries at $z > 1$ from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution*, ApJ, 607, 665

Exercise 1

The surface brightness $\sum$ of an astronomical object is defined as its observed flux divided by its observed angular area:

$$\sum \propto F/(\delta\theta)^2$$

What is $\sum$ as a function of redshift for a class of objects which are both standard candles and standard rulers?

Could the observations of $\sum$ of such class of objects be a useful way for determining the values of cosmological parameters?

Exercise 2

The distance modulus (i.e. difference between the apparent and absolute magnitude) of a type Ia supernova, located at redshift $z = 0.8$, is $\mu = 43.5$. Calculate both luminosity distance and angular diameter distance of this supernova, assuming a spatially flat cosmological model.

Exercise 3

Calculate and plot Hubble diagrams of Supernovae Ia (i.e their distance moduli as a function of their cosmological redshifts) for the following three flat ($\Omega_k = 0$) cosmological models with H_0=71 km/s/Mpc: realistic ($\Omega_M = 0.3$), matter dominated ($\Omega_M = 1$) and dark energy dominated ($\Omega_M = 0$), and compare them with the observed cosmological sample of SN Ia from the Table 5 from Riess et al. 2004, ApJ, 607, 665.
Which cosmological model gives the best agreement?

TABLE 5
MLCS2k2 FULL SAMPLE

SN	z	μ_0 [a]	σ [b]	Host A_V	Sample
SN 1990T........................	0.0400	36.38	0.19	0.37	Gold
SN 1990af.......................	0.050	36.84	0.21	−0.04	Gold
SN 1990O	0.0307	35.90	0.20	0.11	Gold
SN 1991S........................	0.0560	37.31	0.18	0.20	Gold
SN 1991U	0.0331	35.54	0.20	0.37	Gold
SN 1991ag	0.0141	34.13	0.25	0.12	Gold

Note: only a part of the Table 5 from Riess et al. 2004, ApJ, 607, 665 is shown here, and its full version will be provided in the electronic form.

Solution 1

The equation for flux as a function of distance, in cosmology, is

$$F \propto \frac{L}{D_L^2},$$

where L is the luminosity of the source and D_L the luminosity distance. On the other hand, the angular size of an object is given by

$$\delta\theta \propto \frac{d}{D_A},$$

where d is its diameter and D_A the angular distance. The surface brightness is thus given by

$$\sum = \frac{F}{(\delta\theta)^2} \propto \frac{L}{d^2}\left(\frac{D_A}{D_L}\right)^2 = \frac{L}{d^2(1+z)^4}.$$

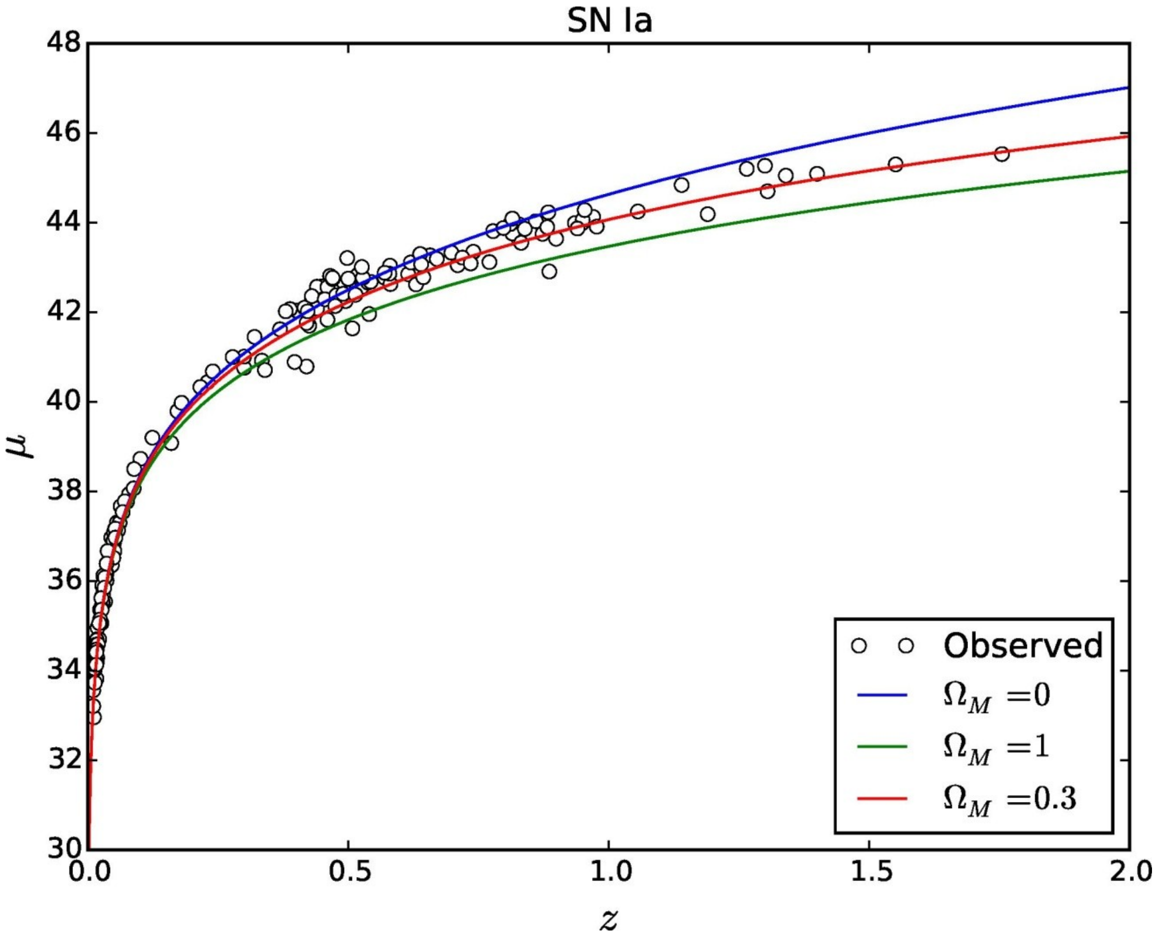
- Surface brightness of such objects is independent of any cosmological model, and therefore cannot be used for determination of cosmological parameters

Solution 2

$$\mathrm{D}_L(z) = 10^{\frac{\mu-25}{5}}\,\mathrm{Mpc} \approx 5012\,\mathrm{Mpc}, \quad D_A(z) = \frac{D_L(z)}{(1+z)^2} \approx 1547\,\mathrm{Mpc}$$

Solution 3

- Distance modulus: $\mu(z) = 5\log D_L(z; H_0, \Omega_M, \Omega_\Lambda) + 25,\ \ \mu = m - M$
- Luminosity distance: $D_L(z; H_0, \Omega_M, \Omega_\Lambda) = \frac{c(1+z)}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_M(1+z')^3 + \Omega_\Lambda}}, \quad \Omega_k = 0$



SN Ia Hubble diagrams obtained by Python script in "SNIa.py"