MASS 2023 Course:

Gravitational Lenses

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Reminder to previous lecture

- **Comoving distance:**

$$D_C(z) = \frac{c}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_M(1+z')^3 + \Omega_k(1+z')^2 + \Omega_\Lambda}}$$

- **Angular diameter distance:**

$$\Omega_k = 0 \Rightarrow D_A(z) = \frac{D_C(z)}{1+z}, \quad D_A(z_1, z_2) = \frac{D_C(z_2) - D_C(z_1)}{1+z_2}$$

- **Luminosity distance:**

$$\Omega_k = 0 \Rightarrow D_L(z) = (1+z)D_C(z) = (1+z)^2 D_A(z)$$

- **Comoving volume:**

$$\Omega_k = 0 \Rightarrow V_C(z) = \frac{4\pi}{3} D_C^3(z)$$

Lecture 04

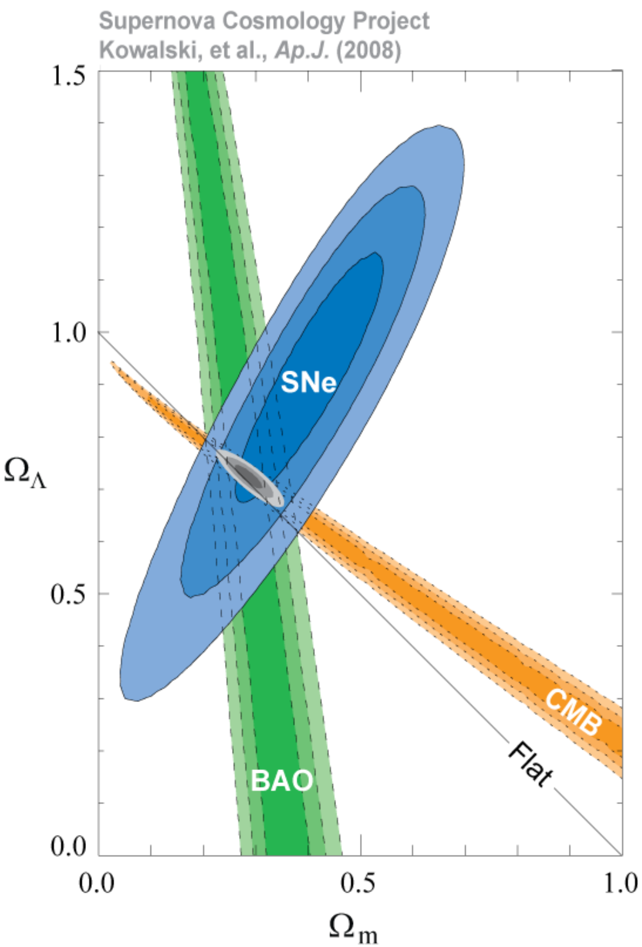
1) Observational cosmology
2) Cosmological tests:
    - **SN Ia**
    - **CMBR**
    - BAO
    - Faint galaxy counts
3) Exercises

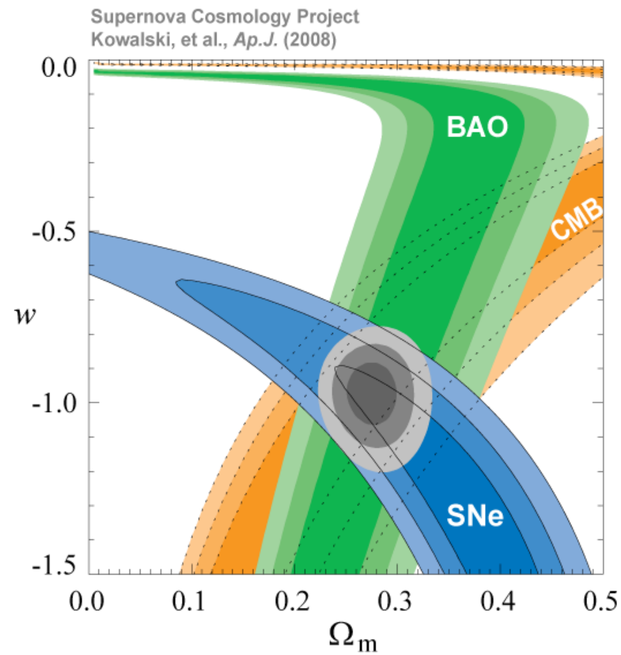
Observational cosmology

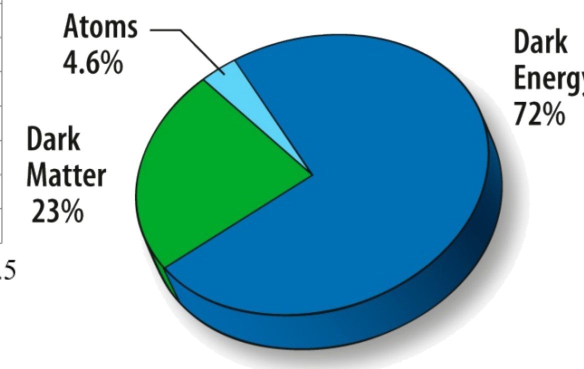
- **Goal:** study of origin, evolution and structure of the Universe using astronomical observations (cosmological tests)
- determination of cosmological parameters
- **Classification of cosmological tests:**

1. **using D_L:** SN Ia, Cepheids in other galaxies, Tully-Fisher and Faber-Jackson relation
2. **using D_A:** CMBR, BAO, GL
3. **using V_C, n or dP:** faint galaxy counts, GL statistics

Parameters of concordance cosmological model







- The most important result of standard ΛCDM cosmological model: more than 95% of present content of the Universe has unknown nature

Supernovae of type Ia

- Originate in close binary systems
- Standard candles because they have the same absolute magnitude *M* which is known (≈ -19^{m}.5)
- Apparent magnitude *m* is measured photometrically
- z is measured spectroscopically
- Relation between the magnitudes:

$$M = m - 5\log\frac{r}{\mathrm{pc}} + 5 \Leftrightarrow$$

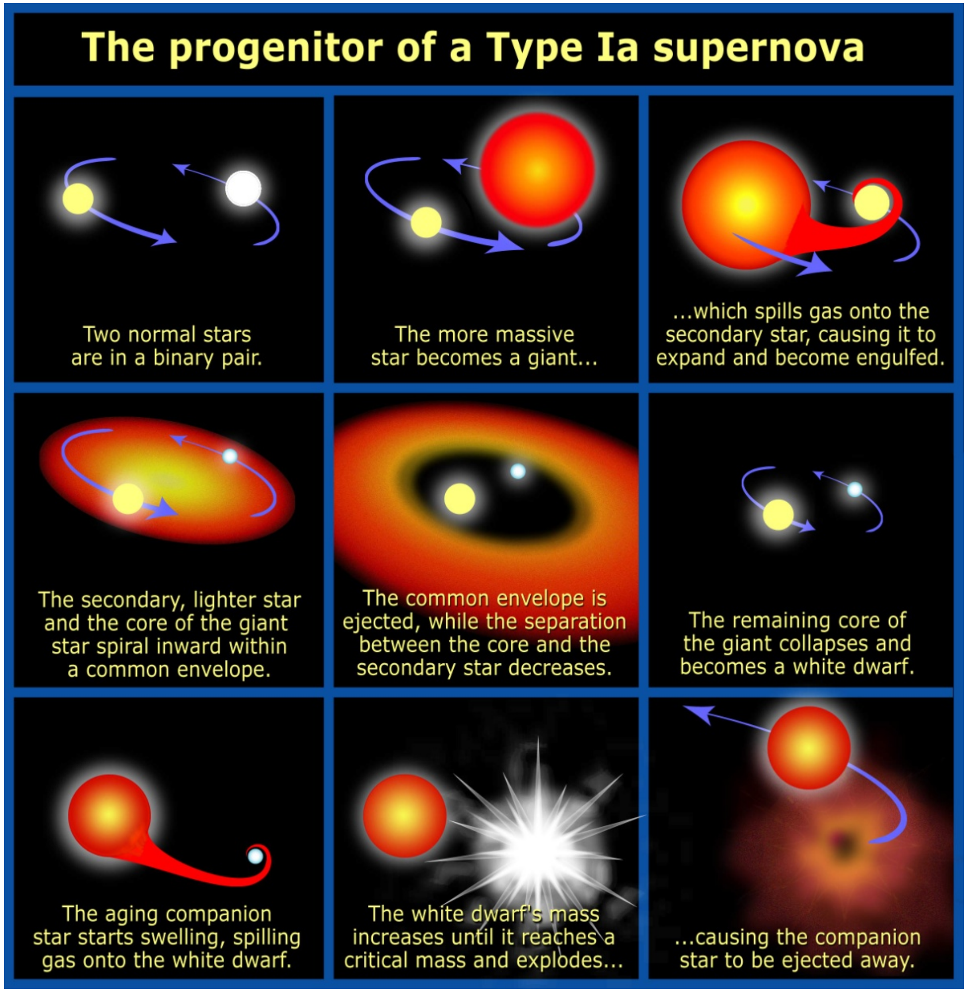
$$M = m - 5\log\frac{r}{\mathrm{Mpc}} - 25 \quad (*)$$

- **Distance modulus**: $\mu = m - M$

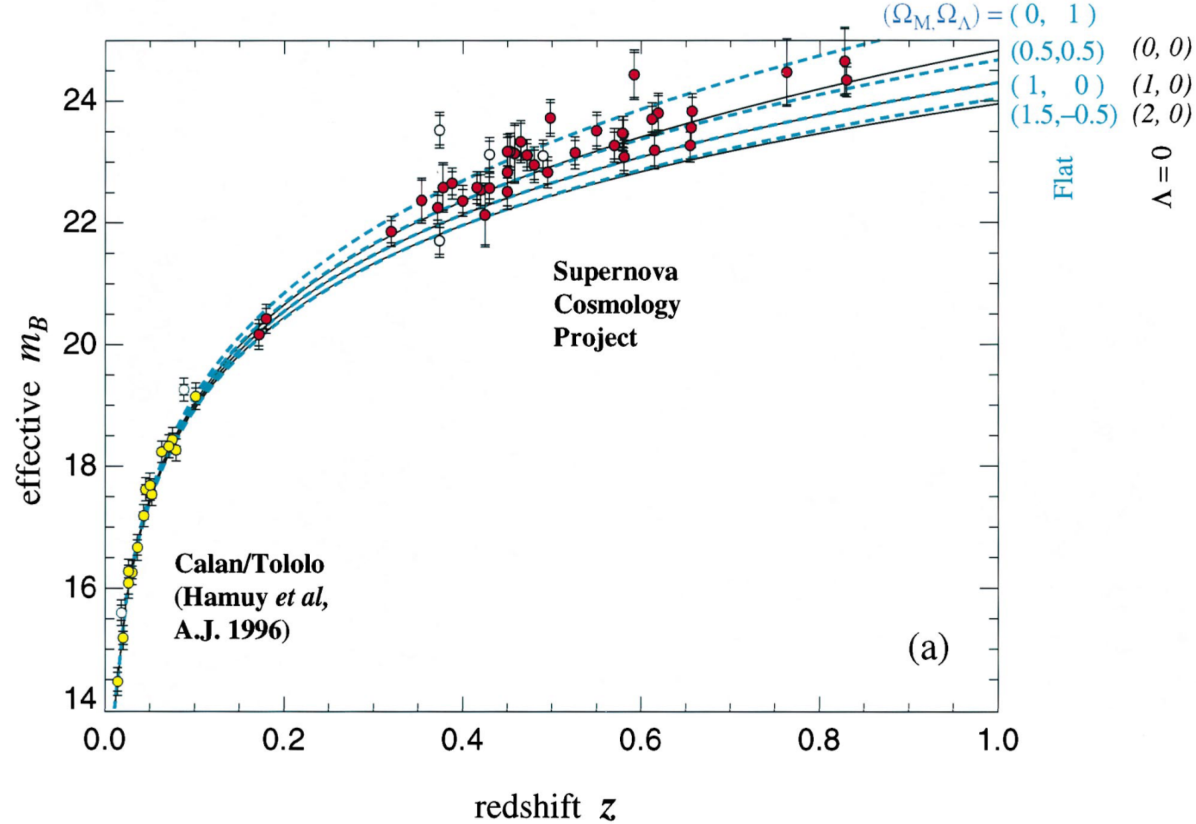
$$r\,[\mathrm{Mpc}] \longmapsto D_L\,[\mathrm{Mpc}] \overset{(*)}{\Rightarrow}$$

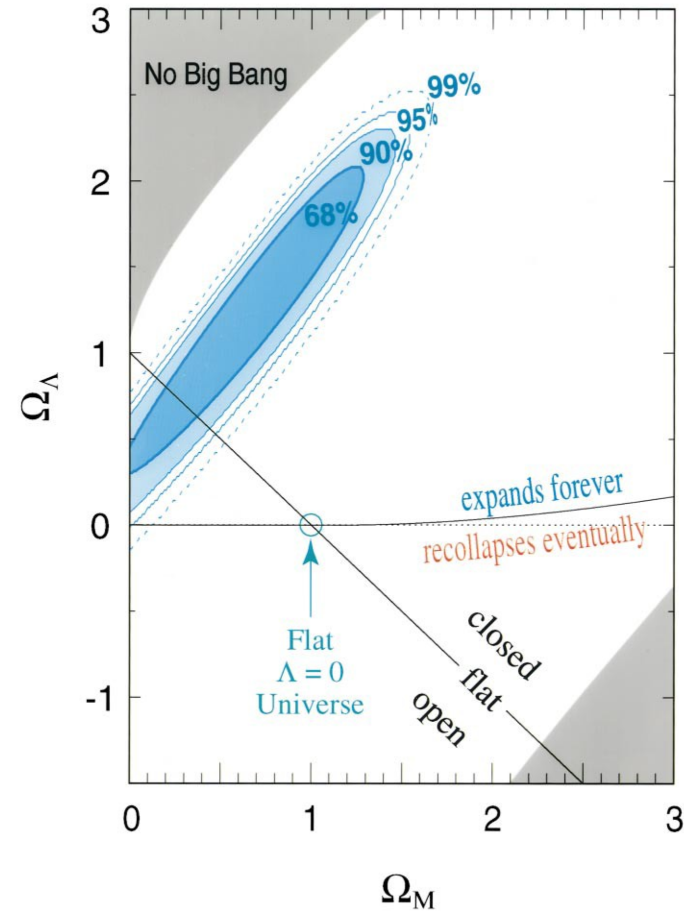
- Cosmological parameters are derived from (Perlmutter et al. ApJ, 1997, 483, 565):

$$\boxed{\mu(z) = 5\log D_L(z; H_0, \Omega_M, \Omega_\Lambda) + 25}$$

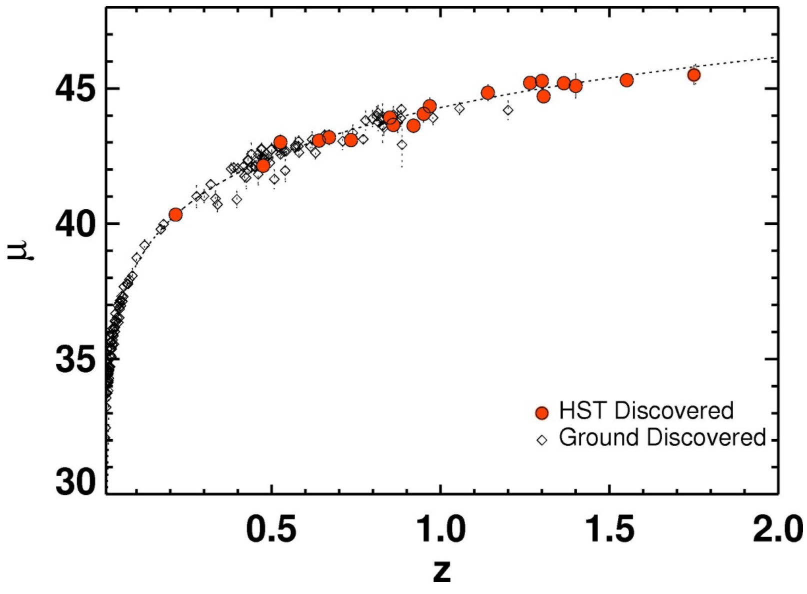


- "Supernova Cosmology Project"
- "High-Z SN Search"





(Perlmutter et al. 1999, ApJ, 517, 565):



(Riess et al. 2004, ApJ, 607, 665)

Finding q_0 from Hubble diagram of SN Ia

- Definitions of the Hubble, deceleration and jerk parameters:

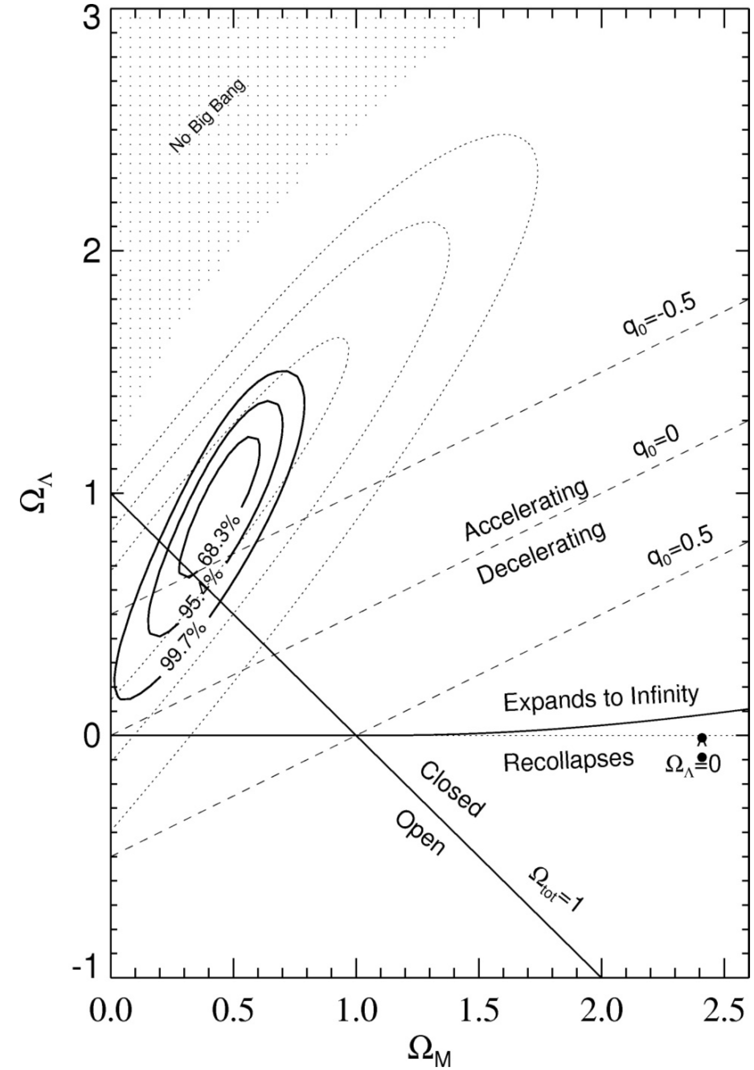
$$H(t) = +\dot{a}/a, \quad q(t) = -\,(\ddot{a}/a)(\dot{a}/a)^{-2},$$

$$j(t) = +(\dddot{a}/a)(\dot{a}/a)^{-3}$$

- Taylor series expansion of $a(t)$ around present epoch t_0:

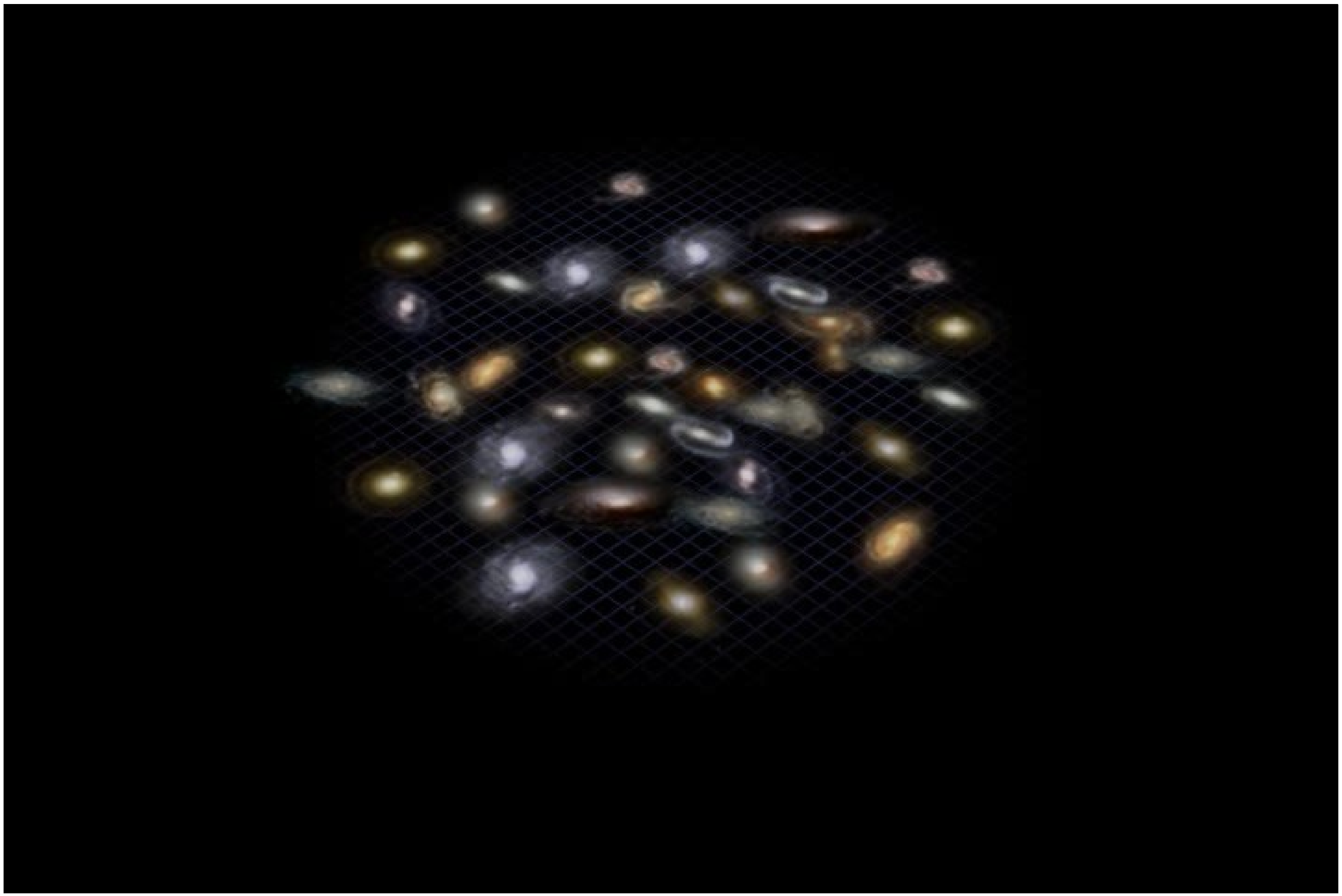
$$a(t) = a_0 \left\{ 1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \frac{1}{3!} j_0 H_0^3 (t - t_0)^3 + O\left[(t - t_0)^4\right] \right\}$$

- Relation between D_L and q_0 in Euclidean space (Riess et al. 2004, ApJ, 607, 665):



$$D_L = c(1+z) \int_0^z \frac{du}{H(u)} = \frac{cz}{H_0} \left[1 + \frac{1}{2}(1 - q_0)z - \frac{1}{6}(1 - q_0 - 3q_0^2 + j_0)z^2 + O(z^3) \right]$$

Accelerating expansion of the Universe: Nobel Prize in physics 2011



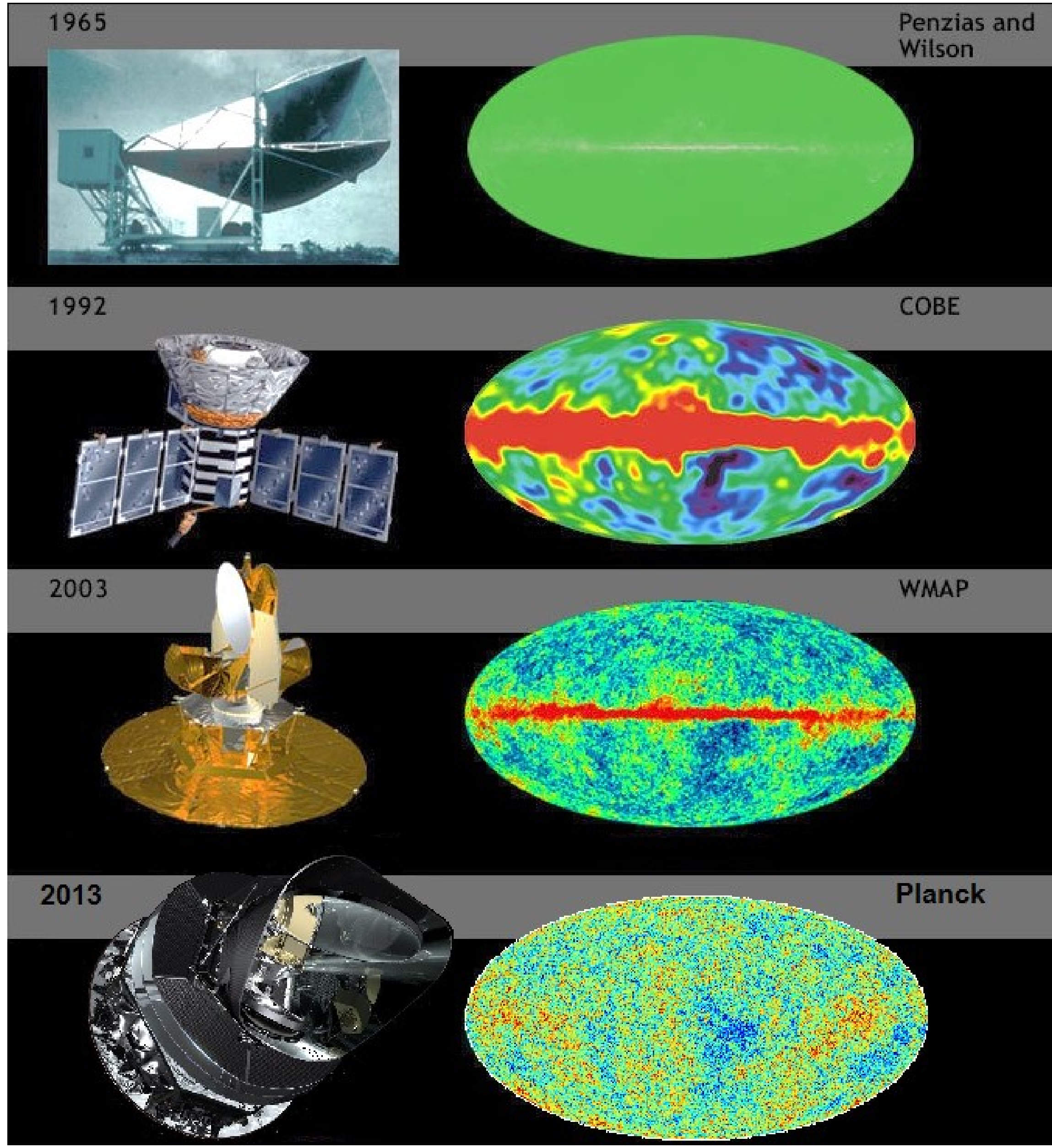
Saul Perlmutter

Brian P. Schmidt

Adam G. Riess

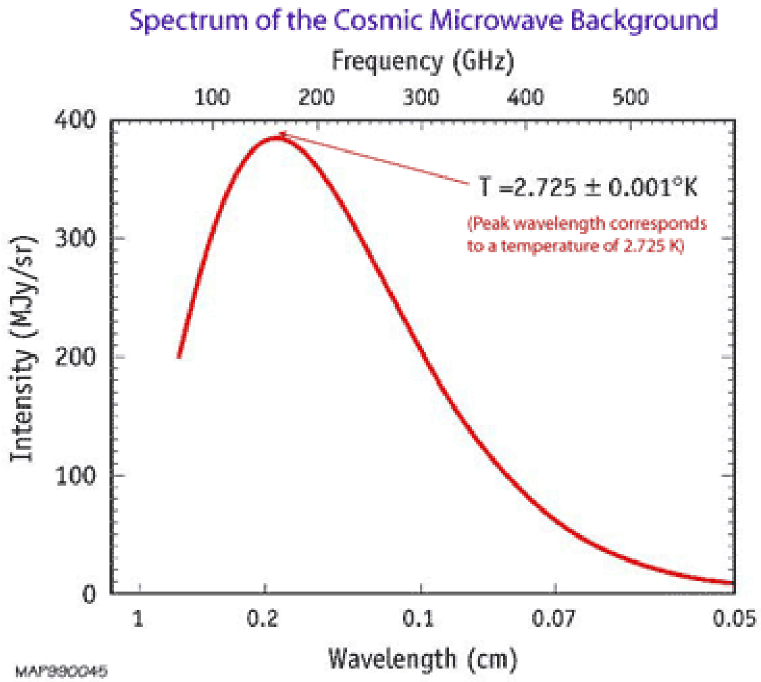
Cosmic Microwave Background Radiation (CMBR)

- Relict radiation from recombination epoch, about 380.000 years after Big Bang ($z \approx 1000$ - 1100)
- Due to cosmic expansion plasma cooled at about 3000 K which enabled recombination of protons and electrons into neutral hydrogen
- These atoms could no longer absorb the thermal radiation and photons starting to travel freely through the space (photon decoupling)
- CMBR temperature that we detect now is $T = 2.725$ K, which is about 1100 times less than in recombination epoch (because of increase of wavelength over time due to expansion of space)

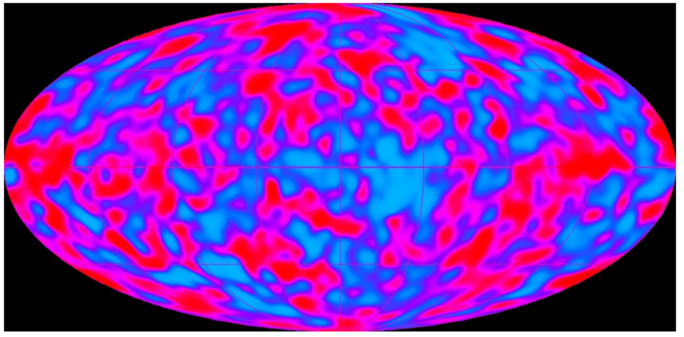


Cosmic Background Explorer (COBE)

- CMBR has Planck spectrum: the most perfect spectrum of black body radiation in nature
- Anisotropy (angular variations) in CMBR temperature due to slight variations in the density of the matter from which the light was last scattered
- These inhomogeneities were caused by quantum fluctuations in the inflaton field that caused the inflation
- Density perturbations in the early Universe act like acoustic waves due to opposite effects of gravitation and pressure: acoustic oscillations

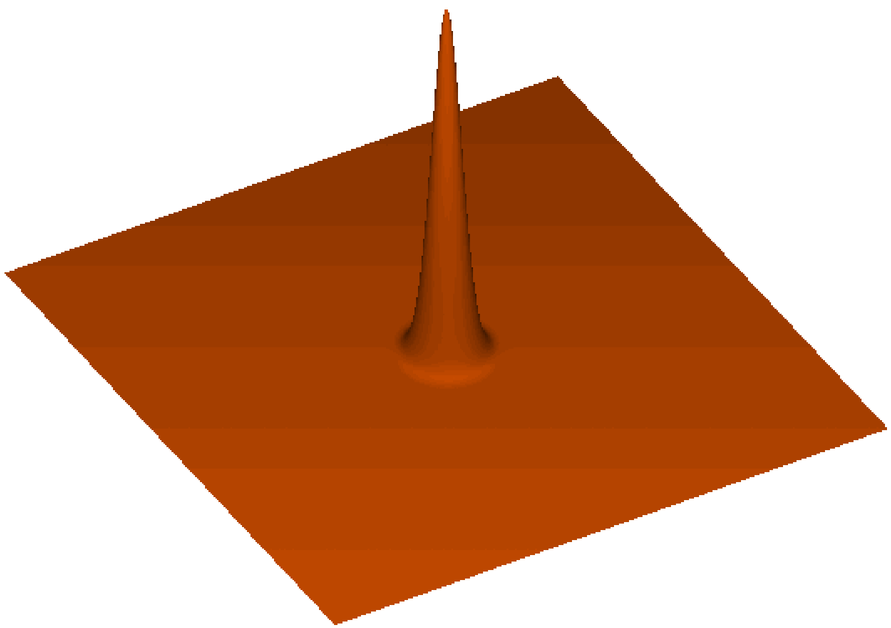


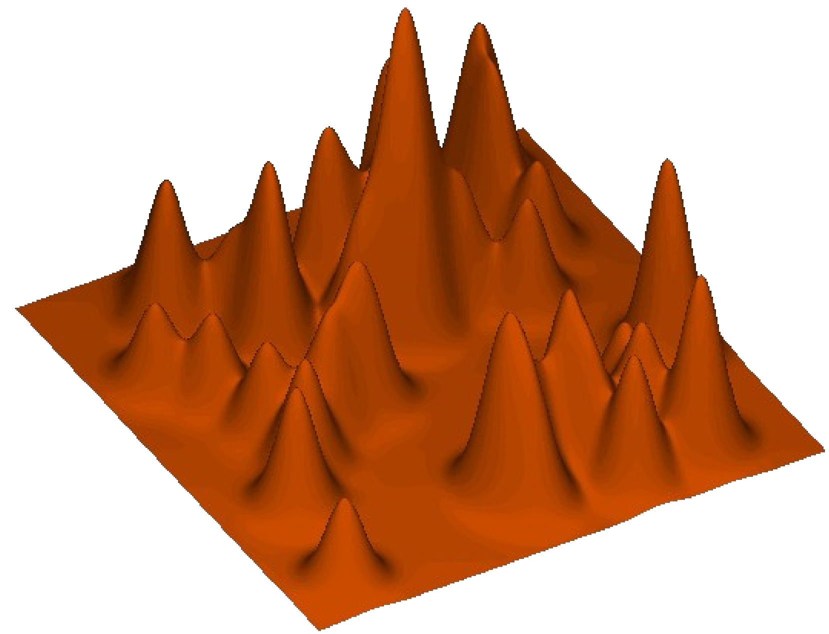
CMBR spectrum obtained by COBE



CMBR fluctuations (Galaxy removed)

Sound waves in the early Universe



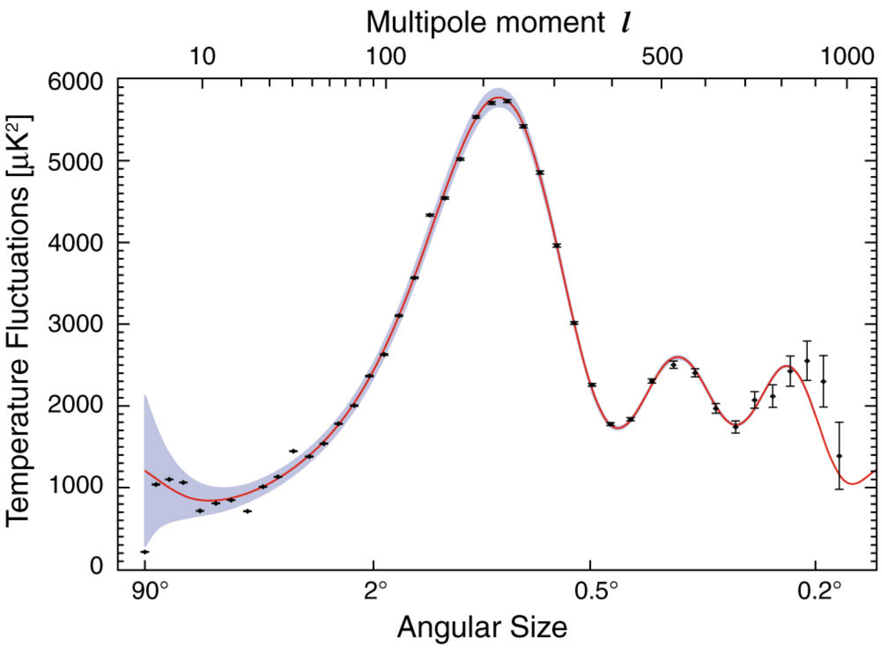


Left: a spherical sound wave from a small overdense region spreads until baryonic matter decouples from radiation. The radius of the acoustic shell (seen as a ring) represents a characteristic length scale: the sound speed times duration of this cosmological epoch

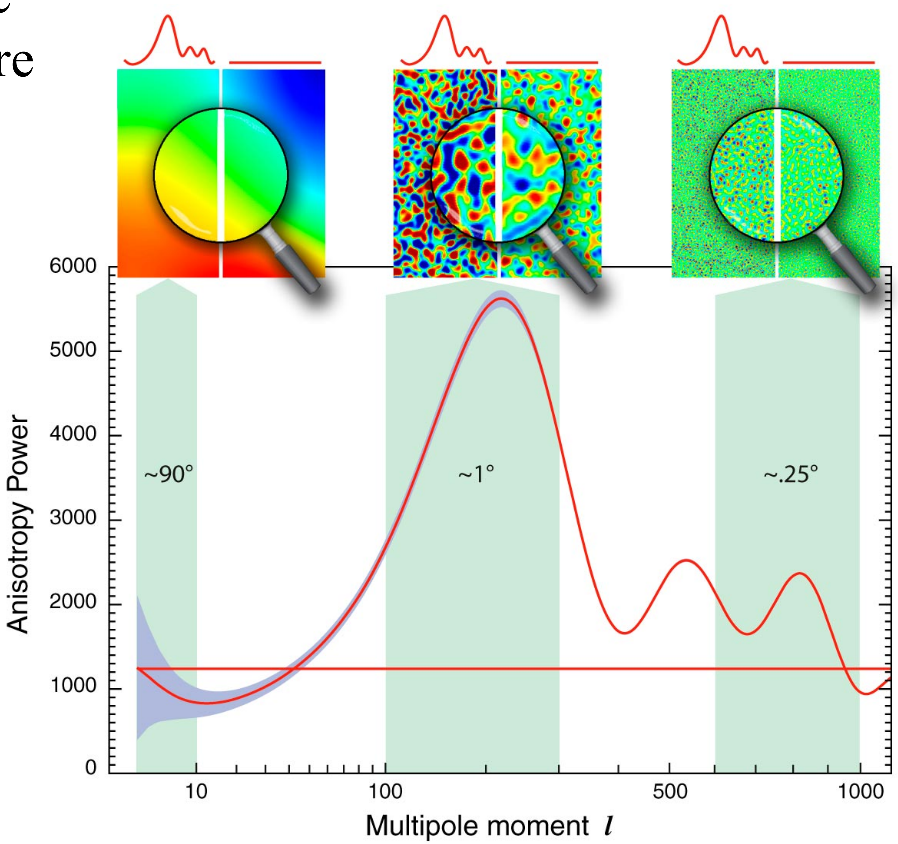
Right: superposition of spherical sound waves in the later universe. Characteristic size can still be extracted from statistical correlations in the large-scale distribution of galaxies.

Wilkinson Microwave Anisotropy Probe (WMAP)

- Order of magnitude of CMBR anisotropy: 10^{-5} K (cosmology as a precise observational science)
- **angular spectrum:** obtained by harmonic analysis of the angular sizes of temperature spots in CMBR (multipole expansion)



Angular spectrum: dependence of temperature fluctuation amplitudes on the angular sizes of the spots



CMBR temperature anisotropy for different angular sizes of the spots

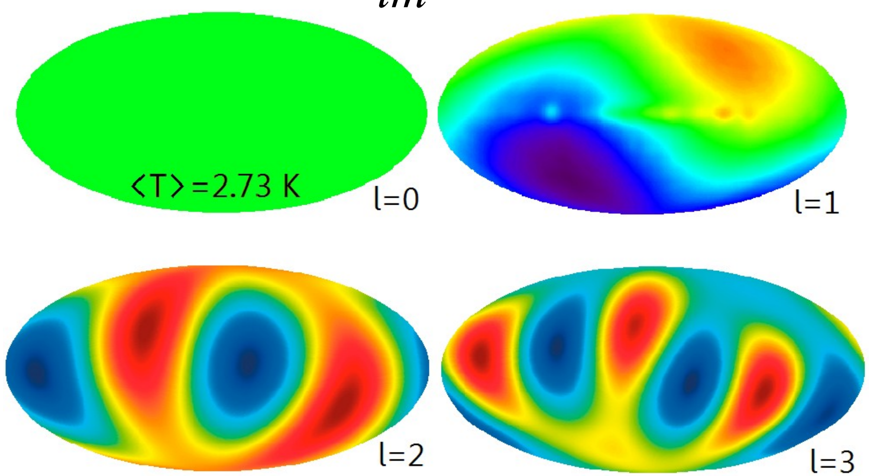
CMBR angular power spectrum

- CMBR anisotropy decomposition to the spherical harmonics $Y_{lm}(\theta, \phi)$:

$$\frac{T(\theta,\phi)-\langle T\rangle}{\langle T\rangle}=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}a_{lm}Y_{lm}(\theta,\phi)\,,\; Y_{lm}=\sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)e^{im\phi}$$

- P_l^m are the Legendre polynomials, harmonic index l is called **multipole**, representing an angular scale in the sky α: $\alpha = \pi / l$, a_{lm} are coefficients:

$$a_{lm}=\int_{\theta=-\pi}^{\pi}\int_{\phi=0}^{2\pi}\frac{T(\theta,\phi)-\langle T\rangle}{\langle T\rangle}Y_{lm}^{*}(\theta,\phi)\,d\Omega$$

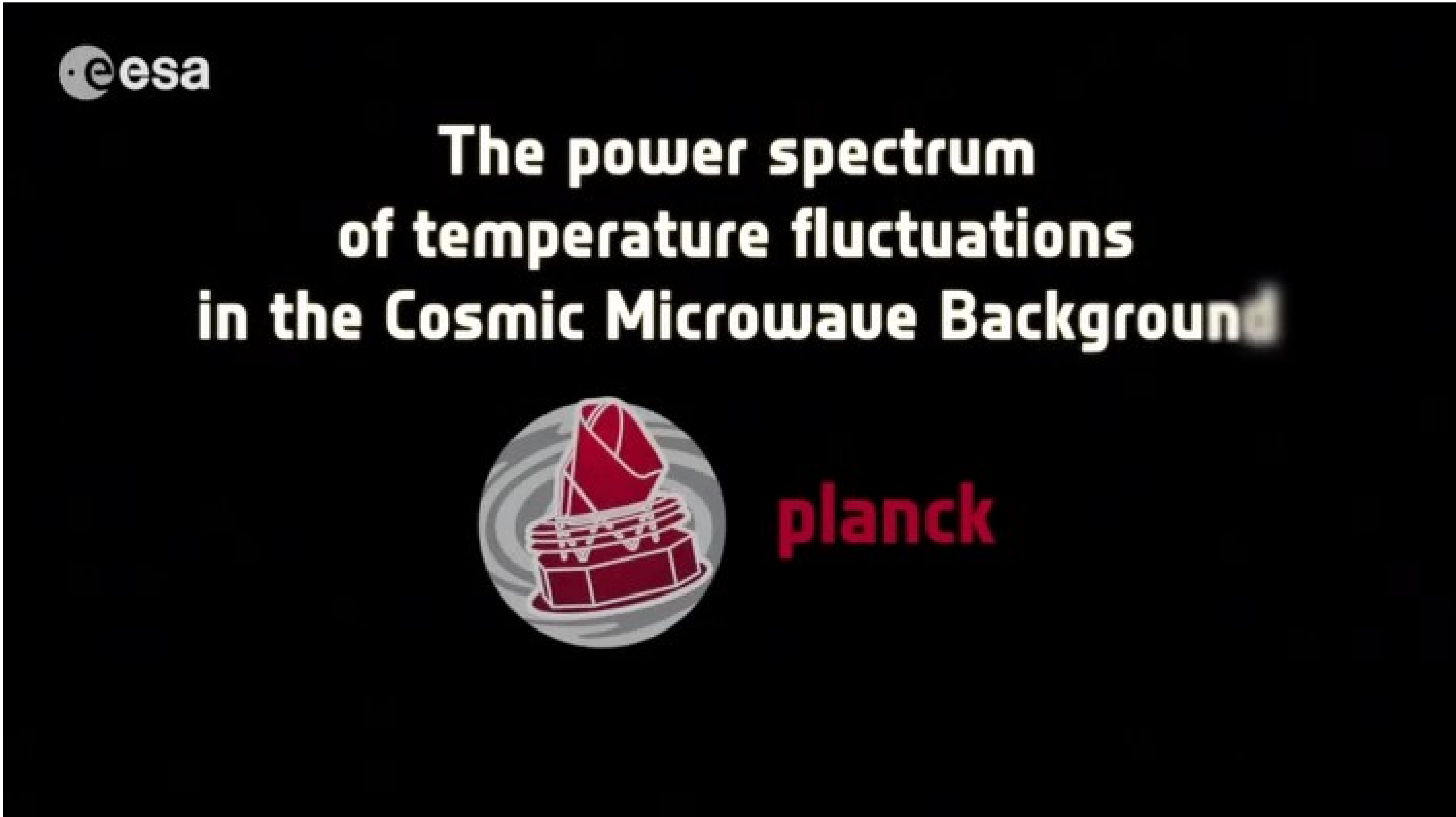


- On small sections of the sky where its curvature can be neglected, the spherical harmonic analysis becomes ordinary 2D Fourier analysis
- **Power spectrum (spectral density)**:

$$C_l=\frac{1}{2l+1}\sum_{m=-l}^{l}\langle|a_{lm}|^2\rangle$$

- C_l is related to the expectation value of the correlation of the temperature between two points in the sky

CMBR angular power spectrum (Planck)

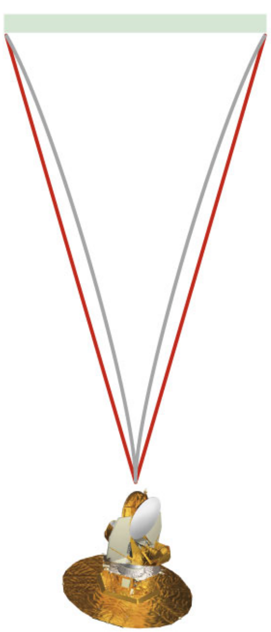


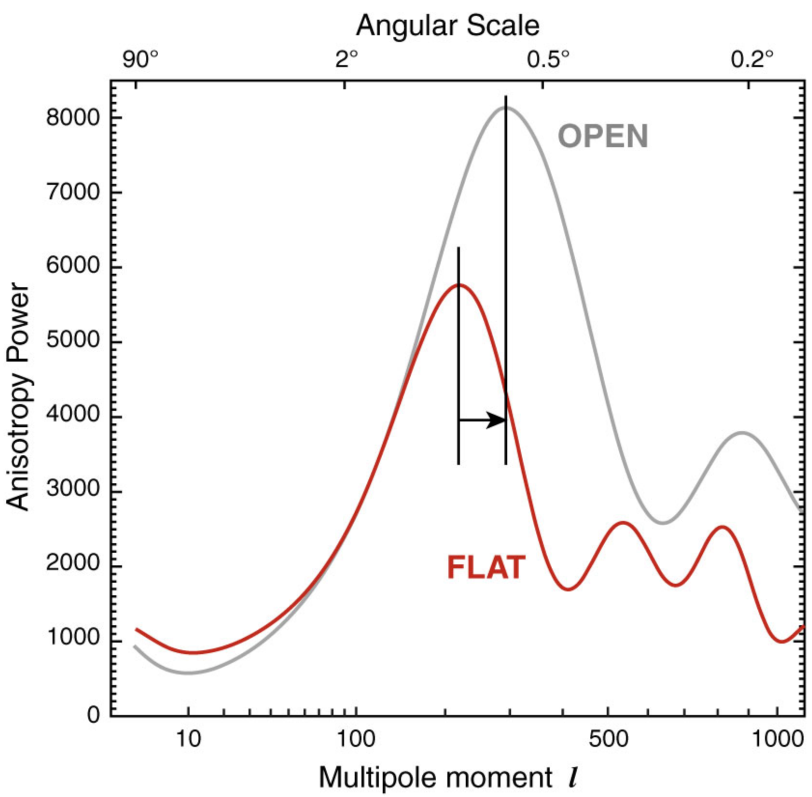
Determination of cosmological parameters from CMBR angular spectrum

- C_l contains all possible information about the underlying density fluctuations which are described by a Gaussian random process

- Cosmological parameters determined from angular scale θ of spots, i.e. from the peak positions l in angular spectrum:

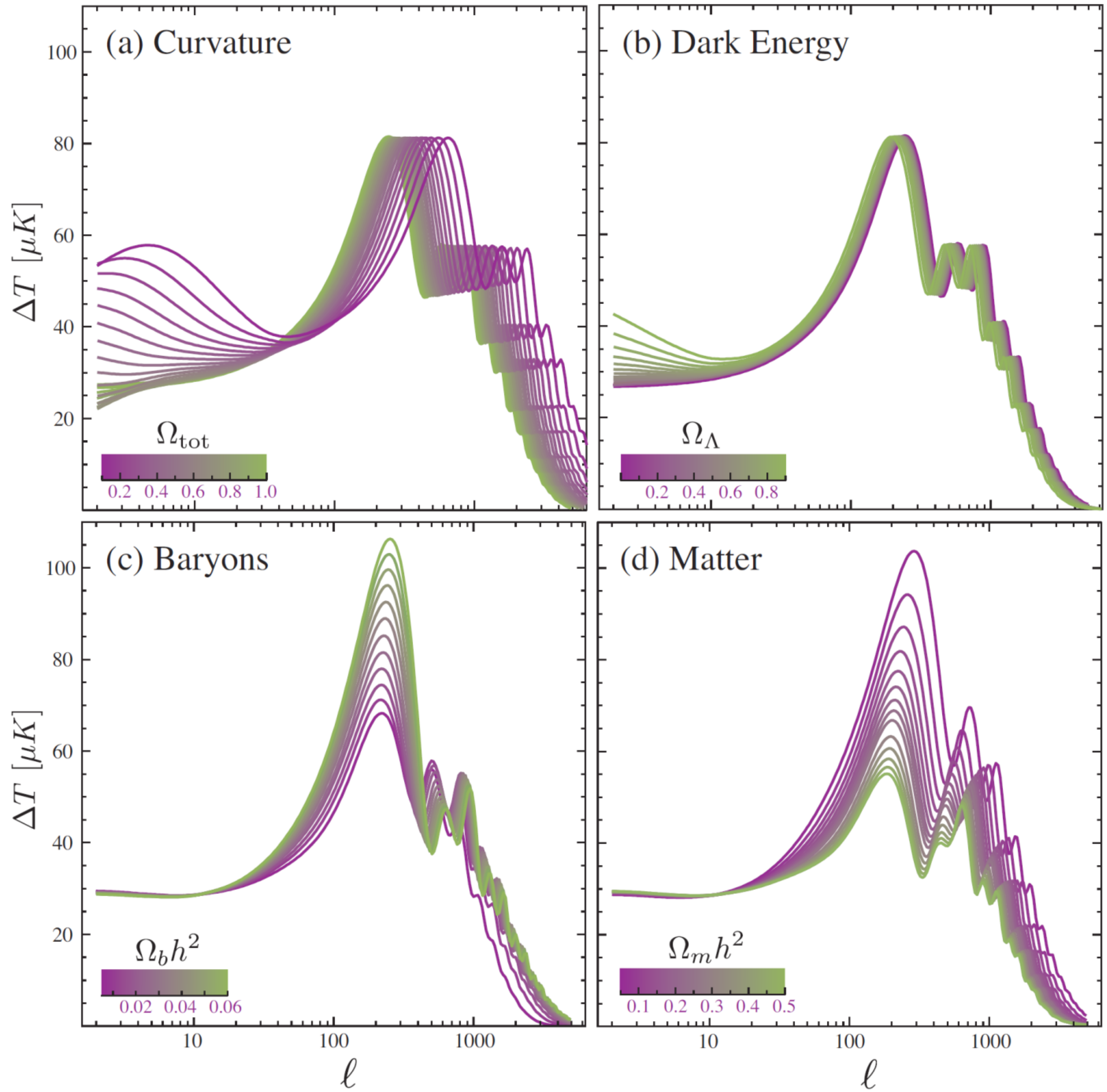
$$l = \frac{\pi}{\theta} = \frac{\pi D_A(z)}{r_s(z)}$$



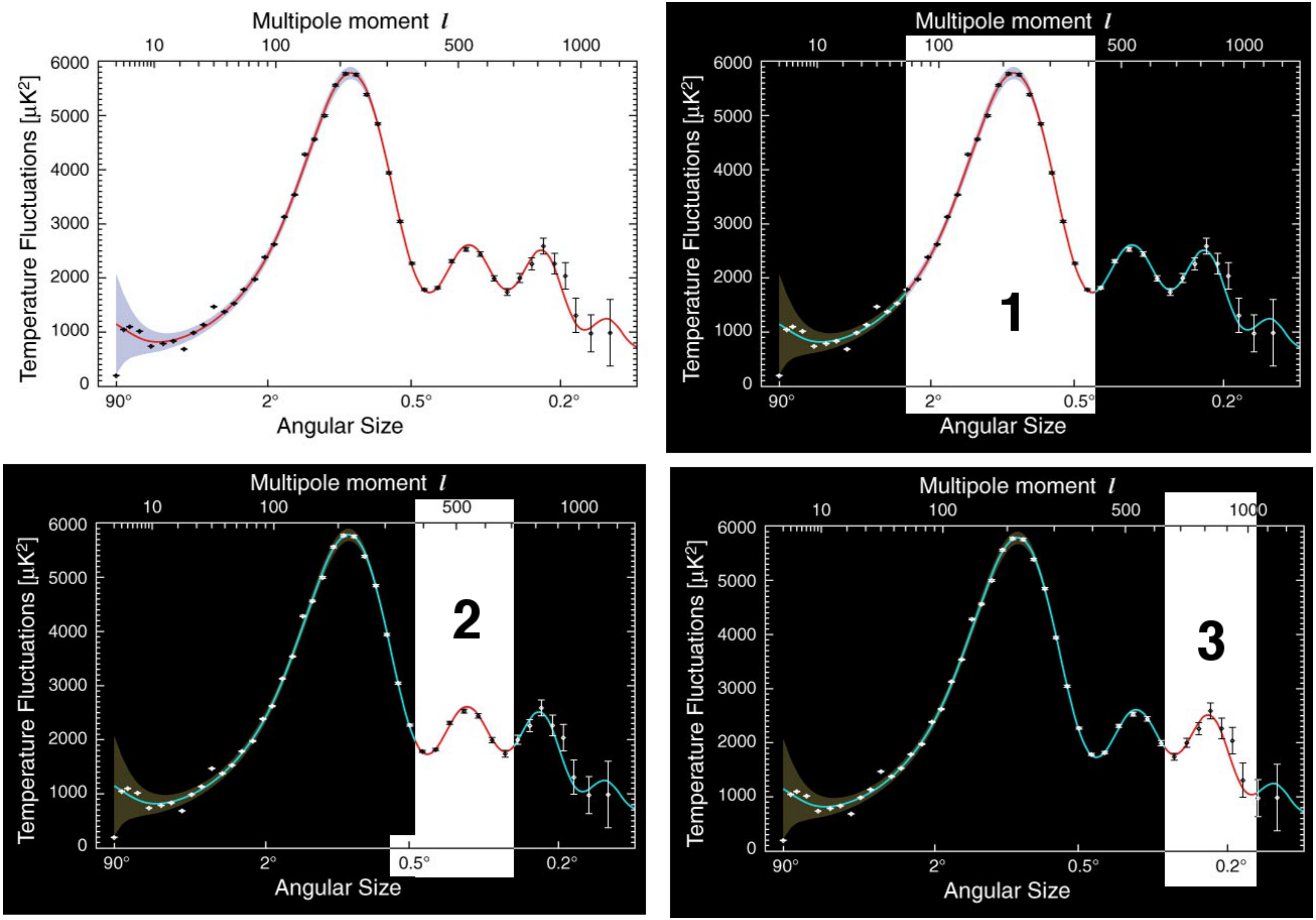


- θ - angular scale
- r_s - sound horizon radius: radius of density perturbation (sound wave) at which the photons decoupled from baryonic matter
- r_s (linear radii of spots) as standard rulers

Dependence of CMBR fluctuations on cosmological model



WMAP power spectrum



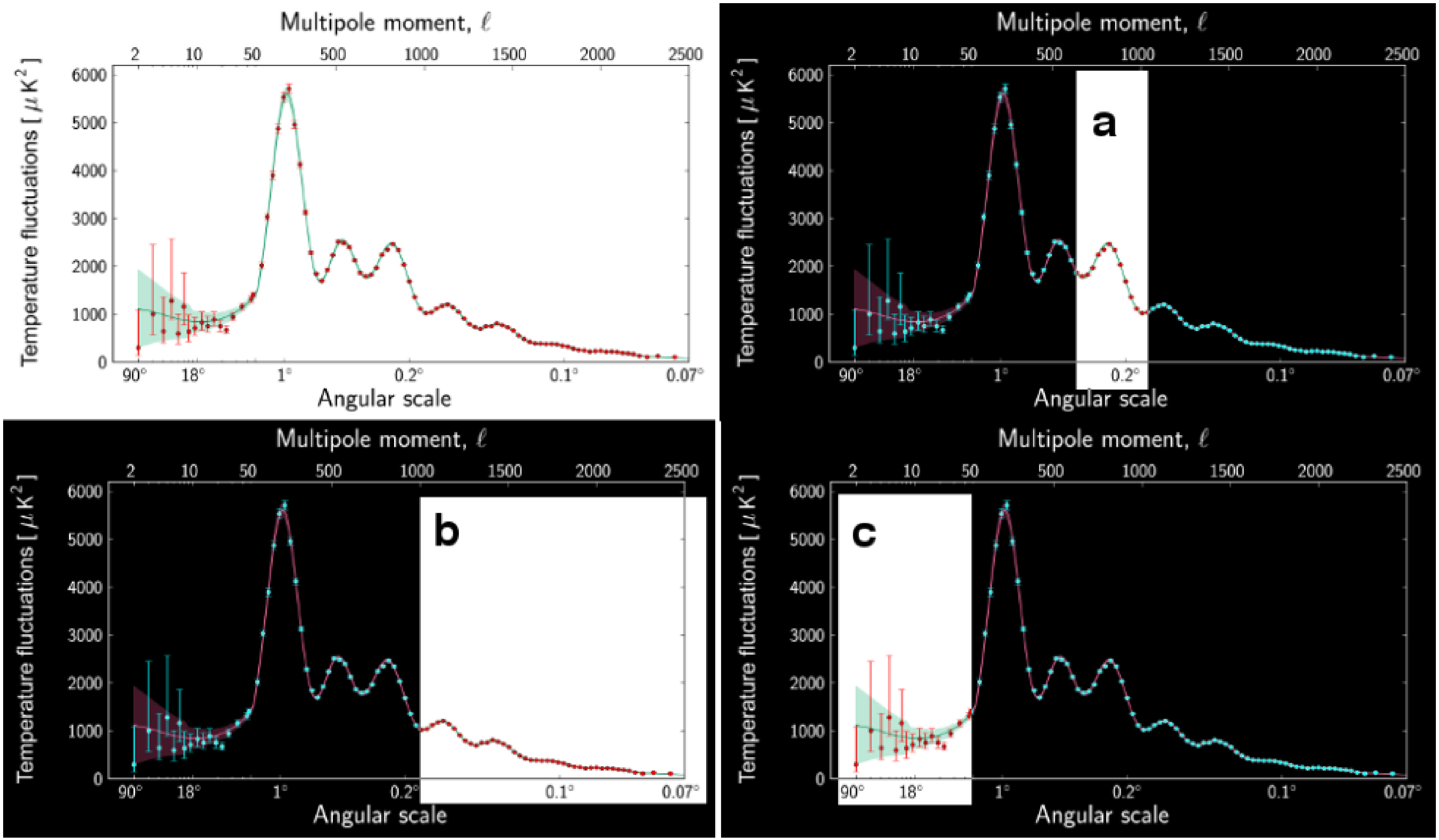
WMAP values of cosmological parameters

WMAP Seven-year Cosmological Parameter Summary

Description	Symbol	*WMAP*-only	*WMAP*+BAO+H_0
Parameters for the Standard ΛCDM Model[a]			
Age of universe	t_0	13.75 ± 0.13 Gyr	13.75 ± 0.11 Gyr
Hubble constant	H_0	71.0 ± 2.5 km s^{-1} Mpc^{-1}	$70.4^{+1.3}_{-1.4}$ km s^{-1} Mpc^{-1}
Baryon density	Ω_b	0.0449 ± 0.0028	0.0456 ± 0.0016
Physical baryon density	$\Omega_b h^2$	$0.02258^{+0.00057}_{-0.00056}$	0.02260 ± 0.00053
Dark matter density	Ω_c	0.222 ± 0.026	0.227 ± 0.014
Physical dark matter density	$\Omega_c h^2$	0.1109 ± 0.0056	0.1123 ± 0.0035
Dark energy density	Ω_Λ	0.734 ± 0.029	$0.728^{+0.015}_{-0.016}$
Curvature fluctuation amplitude, $k_0 = 0.002$ Mpc^{-1}[b]	$\Delta^2_{\mathcal{R}}$	$(2.43 \pm 0.11) \times 10^{-9}$	$(2.441^{+0.088}_{-0.092}) \times 10^{-9}$
Fluctuation amplitude at $8h^{-1}$ Mpc	σ_8	0.801 ± 0.030	0.809 ± 0.024
Scalar spectral index	n_s	0.963 ± 0.014	0.963 ± 0.012
Redshift of matter–radiation equality	$z_{\rm eq}$	3196^{+134}_{-133}	3232 ± 87
Angular diameter distance to matter–radiation eq.[c]	$d_A(z_{\rm eq})$	14281^{+158}_{-161} Mpc	14238^{+128}_{-129} Mpc
Redshift of decoupling	z_*	$1090.79^{+0.94}_{-0.92}$	$1090.89^{+0.68}_{-0.69}$
Age at decoupling	t_*	379164^{+5187}_{-5243} yr	377730^{+3205}_{-3200} yr
Angular diameter distance to decoupling[c,d]	$d_A(z_*)$	14116^{+160}_{-163} Mpc	14073^{+129}_{-130} Mpc
Sound horizon at decoupling[d]	$r_s(z_*)$	$146.6^{+1.5}_{-1.6}$ Mpc	146.2 ± 1.1 Mpc
Acoustic scale at decoupling[d]	$l_A(z_*)$	302.44 ± 0.80	302.40 ± 0.73
Reionization optical depth	τ	0.088 ± 0.015	0.087 ± 0.014
Redshift of reionization	$z_{\rm reion}$	10.5 ± 1.2	10.4 ± 1.2

(Jarosik et al. 2011, ApJS, 192, 14)

Planck power spectrum



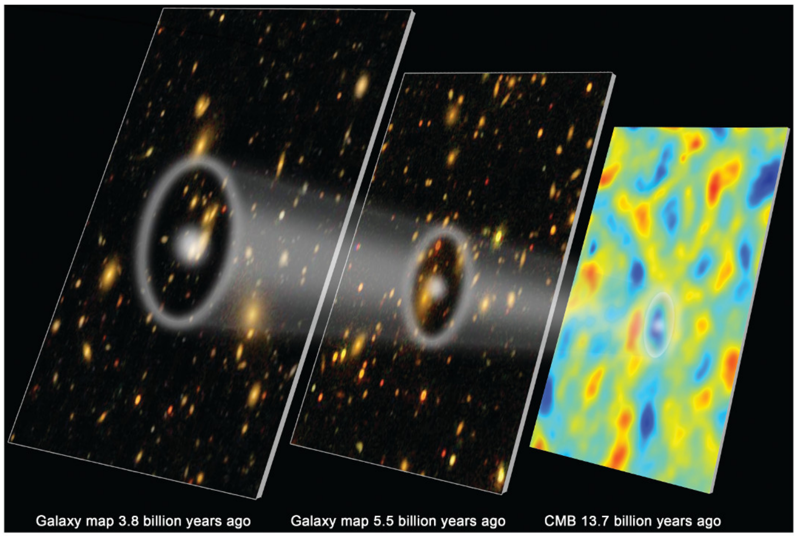
Planck values of cosmological parameters

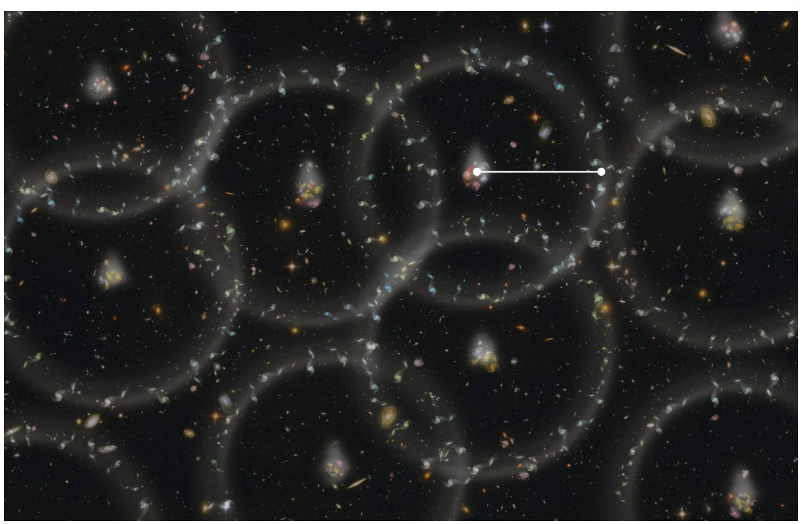
Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022	0.02249 ± 0.00025	0.0240 ± 0.0012	0.02236 ± 0.00015	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1206 ± 0.0021	0.1177 ± 0.0020	0.1158 ± 0.0046	0.1202 ± 0.0014	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04077 ± 0.00047	1.04139 ± 0.00049	1.03999 ± 0.00089	1.04090 ± 0.00031	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0522 ± 0.0080	0.0496 ± 0.0085	0.0527 ± 0.0090	$0.0544^{+0.0070}_{-0.0081}$	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.040 ± 0.016	$3.018^{+0.020}_{-0.018}$	3.052 ± 0.022	3.045 ± 0.016	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9626 ± 0.0057	0.967 ± 0.011	0.980 ± 0.015	0.9649 ± 0.0044	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0 [km s^{-1} Mpc^{-1}] . .	66.88 ± 0.92	68.44 ± 0.91	69.9 ± 2.7	67.27 ± 0.60	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.679 ± 0.013	0.699 ± 0.012	$0.711^{+0.033}_{-0.026}$	0.6834 ± 0.0084	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.321 ± 0.013	0.301 ± 0.012	$0.289^{+0.026}_{-0.033}$	0.3166 ± 0.0084	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1434 ± 0.0020	0.1408 ± 0.0019	$0.1404^{+0.0034}_{-0.0039}$	0.1432 ± 0.0013	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09589 ± 0.00046	0.09635 ± 0.00051	$0.0981^{+0.0016}_{-0.0018}$	0.09633 ± 0.00029	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8118 ± 0.0089	0.793 ± 0.011	0.796 ± 0.018	0.8120 ± 0.0073	0.8111 ± 0.0060	0.8102 ± 0.0060
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$.	0.840 ± 0.024	0.794 ± 0.024	$0.781^{+0.052}_{-0.060}$	0.834 ± 0.016	0.832 ± 0.013	0.825 ± 0.011
$\sigma_8 \Omega_m^{0.25}$	0.611 ± 0.012	0.587 ± 0.012	0.583 ± 0.027	0.6090 ± 0.0081	0.6078 ± 0.0064	0.6051 ± 0.0058
z_{re}	7.50 ± 0.82	$7.11^{+0.91}_{-0.75}$	$7.10^{+0.87}_{-0.73}$	7.68 ± 0.79	7.67 ± 0.73	7.82 ± 0.71
$10^9 A_s$	2.092 ± 0.034	2.045 ± 0.041	2.116 ± 0.047	$2.101^{+0.031}_{-0.034}$	2.100 ± 0.030	2.105 ± 0.030
$10^9 A_s e^{-2\tau}$	1.884 ± 0.014	1.851 ± 0.018	1.904 ± 0.024	1.884 ± 0.012	1.883 ± 0.011	1.881 ± 0.010
Age [Gyr]	13.830 ± 0.037	13.761 ± 0.038	$13.64^{+0.16}_{-0.14}$	13.800 ± 0.024	13.797 ± 0.023	13.787 ± 0.020
z_*	1090.30 ± 0.41	1089.57 ± 0.42	$1087.8^{+1.6}_{-1.7}$	1089.95 ± 0.27	1089.92 ± 0.25	1089.80 ± 0.21
r_* [Mpc]	144.46 ± 0.48	144.95 ± 0.48	144.29 ± 0.64	144.39 ± 0.30	144.43 ± 0.26	144.57 ± 0.22
$100\theta_*$	1.04097 ± 0.00046	1.04156 ± 0.00049	1.04001 ± 0.00086	1.04109 ± 0.00030	1.04110 ± 0.00031	1.04119 ± 0.00029

(Planck Collaboration, Aghanim et al. 2018, arXiv:1807.06209)

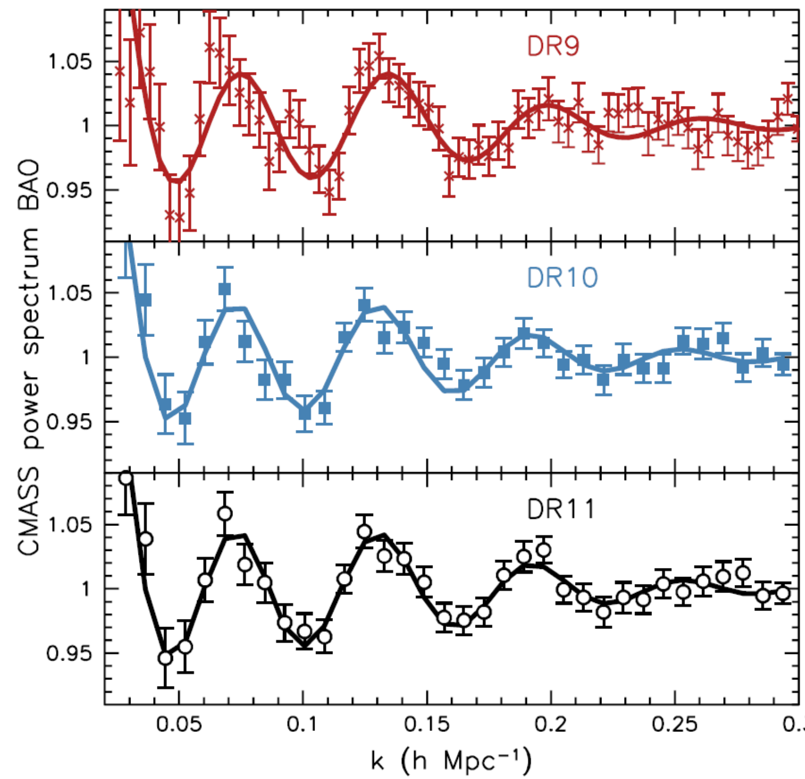
Baryon acoustic oscillations (BAO)

- Imprints of the sound waves from the era of CMBR onto the distribution of galaxies that we see today: galaxies have a tendency to align on the spheres
- Characteristic distance between galaxies provides a standard ruler





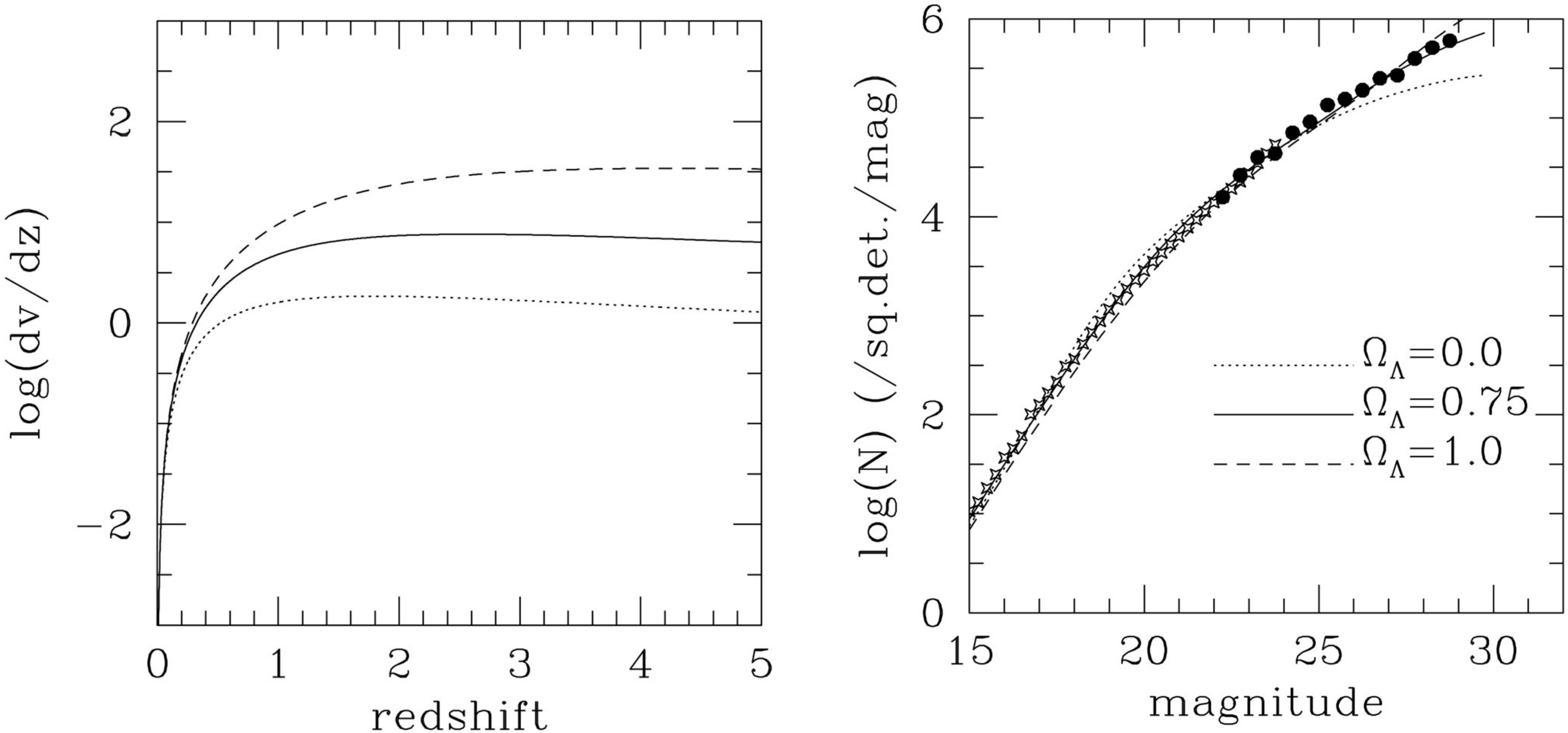
Anderson et al. 2014, MNRAS, 441, 24



$$\theta = \frac{\pi}{l} = \frac{\pi}{k(1+z)D_A(z)}$$

- θ - angular scale
- l - multipole
- k - wavenumber

Number counts of faint galaxies



Left: incremental volume (in units of Hubble volume) per incremental redshift as a function of redshift for three flat cosmological models

Right: comparison between the observed faint galaxy number counts (ground based - open stars; Hubble Deep Fields - solid circles) with the predictions of three flat cosmological models

Exam questions

1. SN Ia as cosmological test
2. CMBR as cosmological test

Literature

- **Textbook:**
  1. Weinberg, S. 2008, *Cosmology*, Oxford University Press Inc., New York, USA
- **Articles:**

  1. Planck Collaboration, 2020, *Planck 2018 results. V. CMB power spectra and likelihoods*, A&A, 641, A5
  2. Planck Collaboration 2020, *Planck 2018 results. VI. Cosmological parameters*, A&A, 641, A6
  3. Perlmutter, S. et al. 1997, *Measurements of the Cosmological Parameters Ω and Λ from the First Seven Supernovae at* $z \geq 0.35$, ApJ, 483, 565
  4. Perlmutter, S. et al. 1999, *Measurements of Ω and Λ from 42 High-Redshift Supernovae*, ApJ, 517, 565
  5. Riess et al. 2004, *Type Ia Supernova Discoveries at* $z > 1$ *from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution*, ApJ, 607, 665

Exercise 1

The surface brightness $\sum$ of an astronomical object is defined as its observed flux divided by its observed angular area:

$$\sum \propto F/(\delta\theta)^2$$

What is $\sum$ as a function of redshift for a class of objects which are both standard candles and standard rulers?

Could the observations of $\sum$ of such class of objects be a useful way for determining the values of cosmological parameters?

Exercise 2

The distance modulus (i.e. difference between the apparent and absolute magnitude) of a type Ia supernova, located at redshift $z = 0.8$, is $\mu = 43.5$. Calculate both luminosity distance and angular diameter distance of this supernova, assuming a spatially flat cosmological model.

Exercise 3

Calculate and plot Hubble diagrams of Supernovae Ia (i.e their distance moduli as a function of their cosmological redshifts) for the following three flat ($\Omega_k = 0$) cosmological models with H_0=71 km/s/Mpc: realistic ($\Omega_M = 0.3$), matter dominated ($\Omega_M = 1$) and dark energy dominated ($\Omega_M = 0$), and compare them with the observed cosmological sample of SN Ia from the Table 5 from Riess et al. 2004, ApJ, 607, 665.

Which cosmological model gives the best agreement?

TABLE 5

MLCS2k2 FULL SAMPLE

SN	z	μ_0 [a]	σ [b]	Host A_V	Sample
SN 1990T......................	0.0400	36.38	0.19	0.37	Gold
SN 1990af......................	0.050	36.84	0.21	−0.04	Gold
SN 1990O	0.0307	35.90	0.20	0.11	Gold
SN 1991S......................	0.0560	37.31	0.18	0.20	Gold
SN 1991U	0.0331	35.54	0.20	0.37	Gold
SN 1991ag	0.0141	34.13	0.25	0.12	Gold

Note: only a part of the Table 5 from Riess et al. 2004, ApJ, 607, 665 is shown here, and its full version will be provided in the electronic form.

Solution 1

The equation for flux as a function of distance, in cosmology, is

$$F \propto \frac{L}{D_L^2},$$

where L is the luminosity of the source and D_L the luminosity distance. On the other hand, the angular size of an object is given by

$$\delta\theta \propto \frac{d}{D_A},$$

where d is its diameter and D_A the angular distance. The surface brightness is thus given by

$$\sum = \frac{F}{(\delta\theta)^2} \propto \frac{L}{d^2}\left(\frac{D_A}{D_L}\right)^2 = \frac{L}{d^2(1+z)^4}.$$

- Surface brightness of such objects is independent of any cosmological model, and therefore cannot be used for determination of cosmological parameters

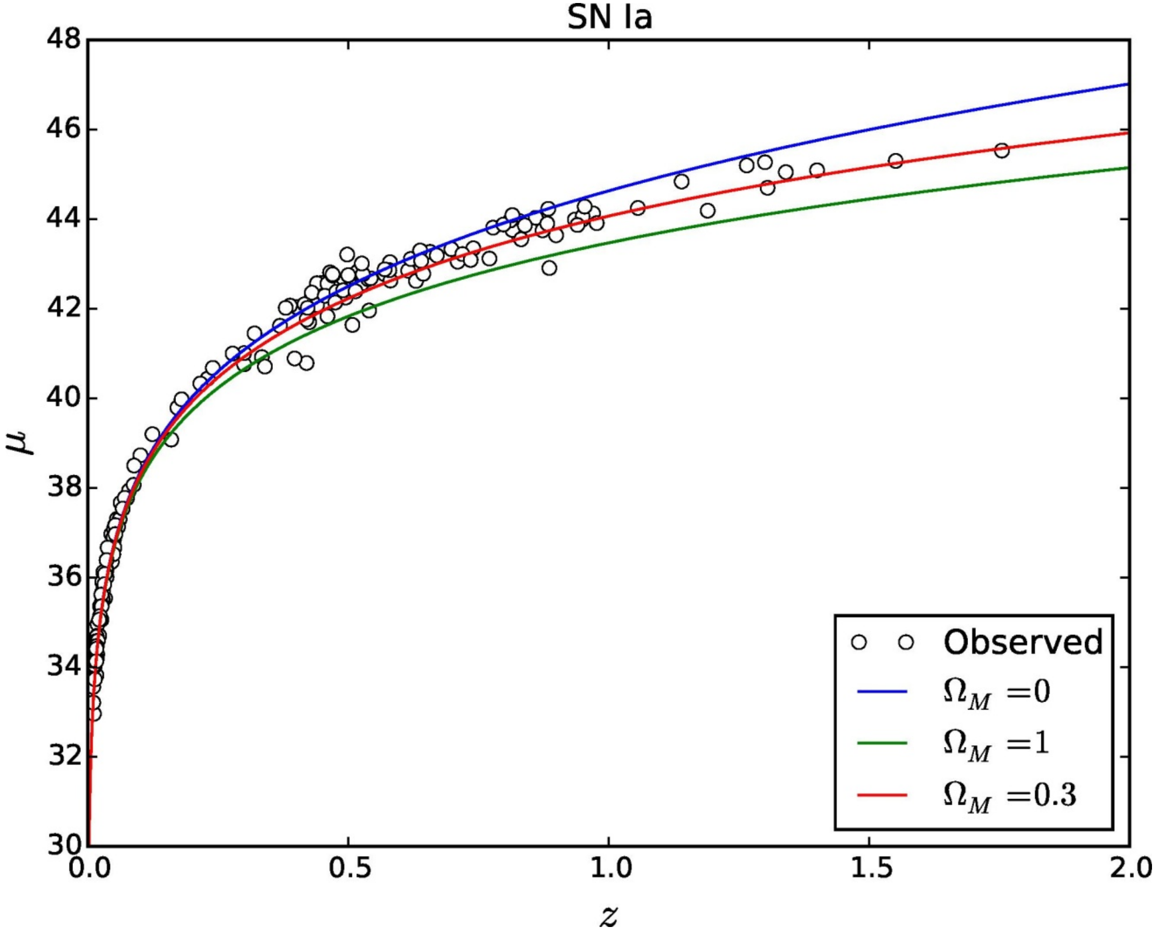
Solution 2

$$\mathrm{D}_L(z) = 10^{\frac{\mu-25}{5}}\,\mathrm{Mpc} \approx 5012\,\mathrm{Mpc}, \quad D_A(z) = \frac{D_L(z)}{(1+z)^2} \approx 1547\,\mathrm{Mpc}$$

Solution 3

- Distance modulus: $\mu(z) = 5\log D_L(z; H_0, \Omega_M, \Omega_\Lambda) + 25,\ \mu = m - M$

- Luminosity distance: $D_L(z; H_0, \Omega_M, \Omega_\Lambda) = \frac{c(1+z)}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_M(1+z')^3 + \Omega_\Lambda}}, \quad \Omega_k = 0$



SN Ia Hubble diagrams obtained by Python script "SNIa.py"