MASS 2023 Course:

Gravitational Lenses

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Lecture 08

1. Fermat potential and Fermat principle
2. Lensing time delay
3. Measuring lensing time delay from the observed light curves of gravitationally lensed quasars
4. Determination of H_0 from the measured time-delays of gravitationally lensed quasars
5. Exercises

Fermat potential

- Lens equation can be rewritten like this:
  $$\vec{\beta} = \vec{\theta} - \vec{\nabla}\psi(\theta) \Leftrightarrow (\vec{\theta} - \vec{\beta}) - \vec{\nabla}\psi(\vec{\theta}) = 0 \Leftrightarrow \vec{\nabla}\left[\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta})\right] = 0,$$
  where ψ is deflection potential
- **Fermat potential** is the following function: $\boxed{\tau(\vec{\theta}, \vec{\beta}) = \frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta})}$
- Lens equation is then: $\vec{\nabla}\tau(\vec{\theta}, \vec{\beta}) = 0$
- Images appear at locations that correspond to extrema or saddle points in Fermat potential

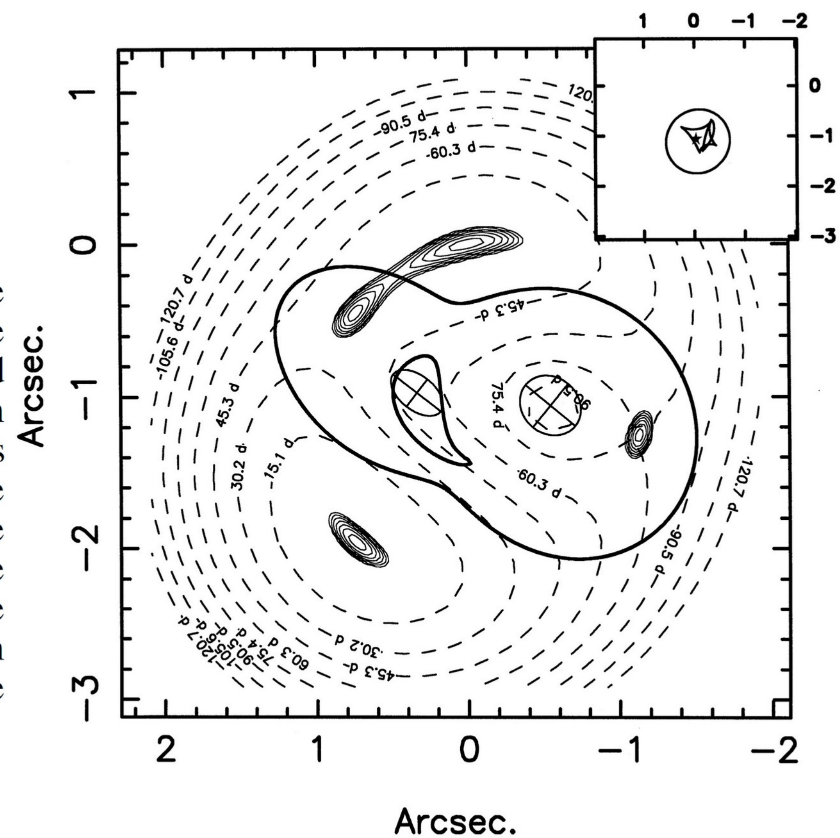


FIG. 4. The lensing model for CLASS B1608 + 656, showing the image positions (light solid contours), locations and ellipticities of the lens components (ellipses), lens isopotential contours (dark solid contours), and surfaces of constant time delay (dashed contours, with delays labeled). Note that as in all quad systems, the observed images correspond to minima or saddle-points in the Fermat potential, while the unobserved fifth image that appears at a maximum of the time delay is highly demagnified. The *Inset* at the upper right shows the source-plane configuration along with the outer critical curves (the transition from one to three total images) and the inner caustics (from three to five images). The star marks the inferred location of the background radio source, which lies just inside the inner caustic.

Myers S. T. PNAS 1999, 96, 4236

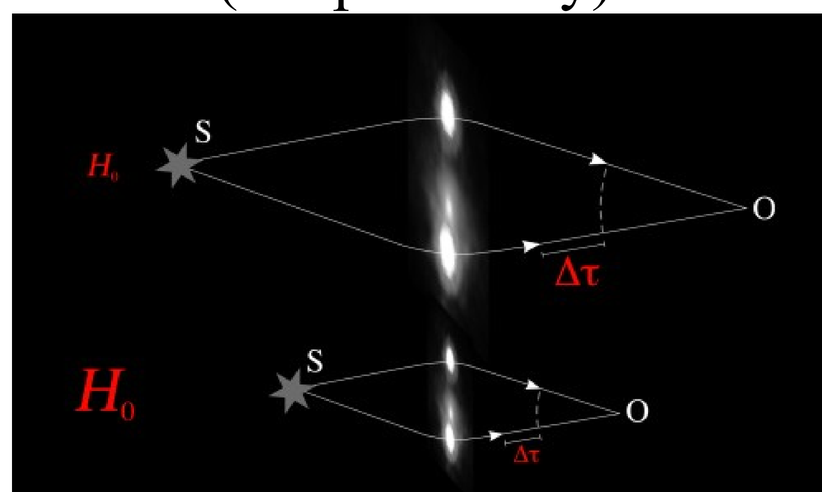
Lensing time delay

- Physical meaning of **Fermat potential**: it is, up to an affine transformation, the **light travel time** along a ray starting at position $\vec{\beta}$, traversing the lens plane at position $\vec{\theta}$ and arriving at the observer
- **Fermat principle:** the physical light rays are those for which the light travel time is stationary
- **Light travel time (**physical time delay function**)** for a lensed image:

$$\boxed{\tau(\vec{\theta},\vec{\beta}) = \tau_{\rm geom} + \tau_{\rm grav} = \frac{D_{\Delta t}}{c}\left(\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \psi(\vec{\theta})\right), \quad D_{\Delta t} = (1+z_d)\frac{D_d D_s}{D_{ds}},}$$

where $\tau_{\rm geom}$ is extra path length between observer and source, while $\tau_{\rm grav}$ is retardation due to gravitational potential (Shapiro delay)

- $D_{\Delta t}$ is the **time-delay distance** which is $\propto H_0^{-1}$ and very weakly sensitive to Ω_M and Ω_Λ
- $\Delta t = \tau_2 - \tau_1 \Rightarrow D_{\Delta t} \Rightarrow H_0$
- Estimates depend on lens model (ψ)



Measuring the time delays from the light curves of the images

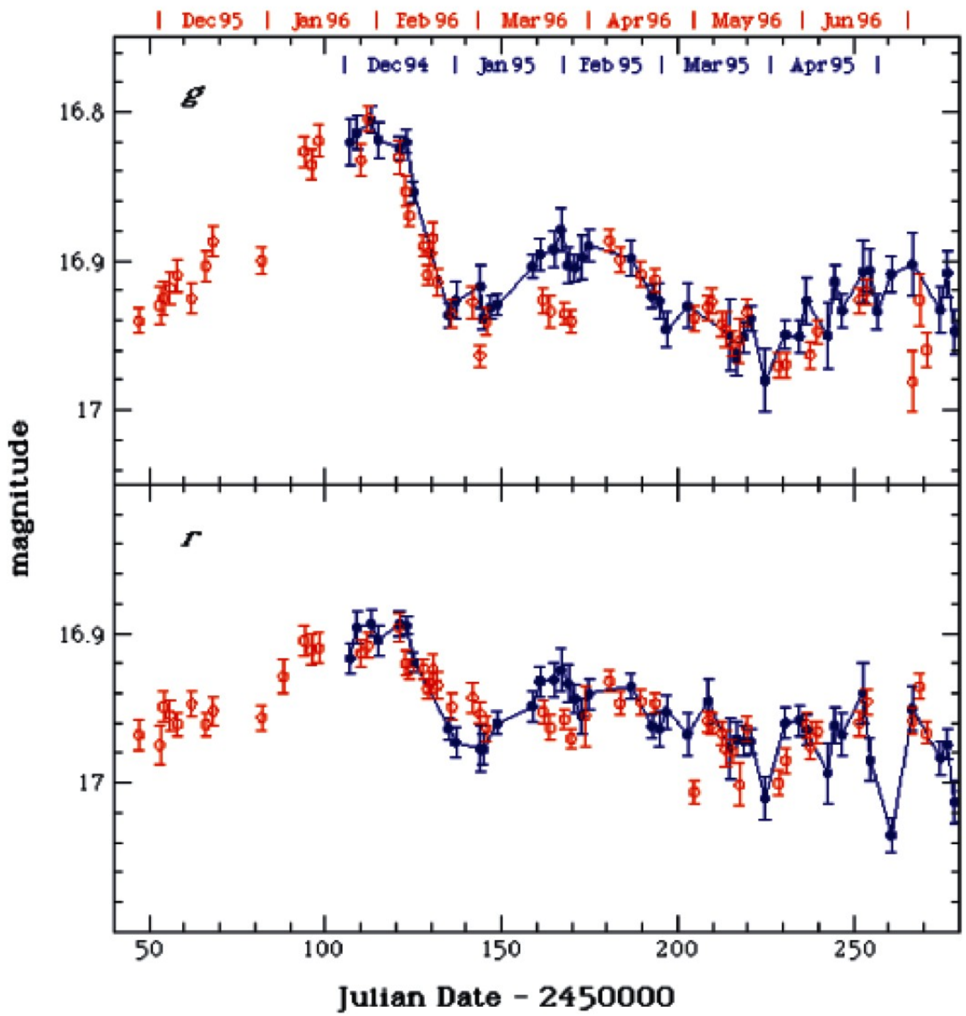
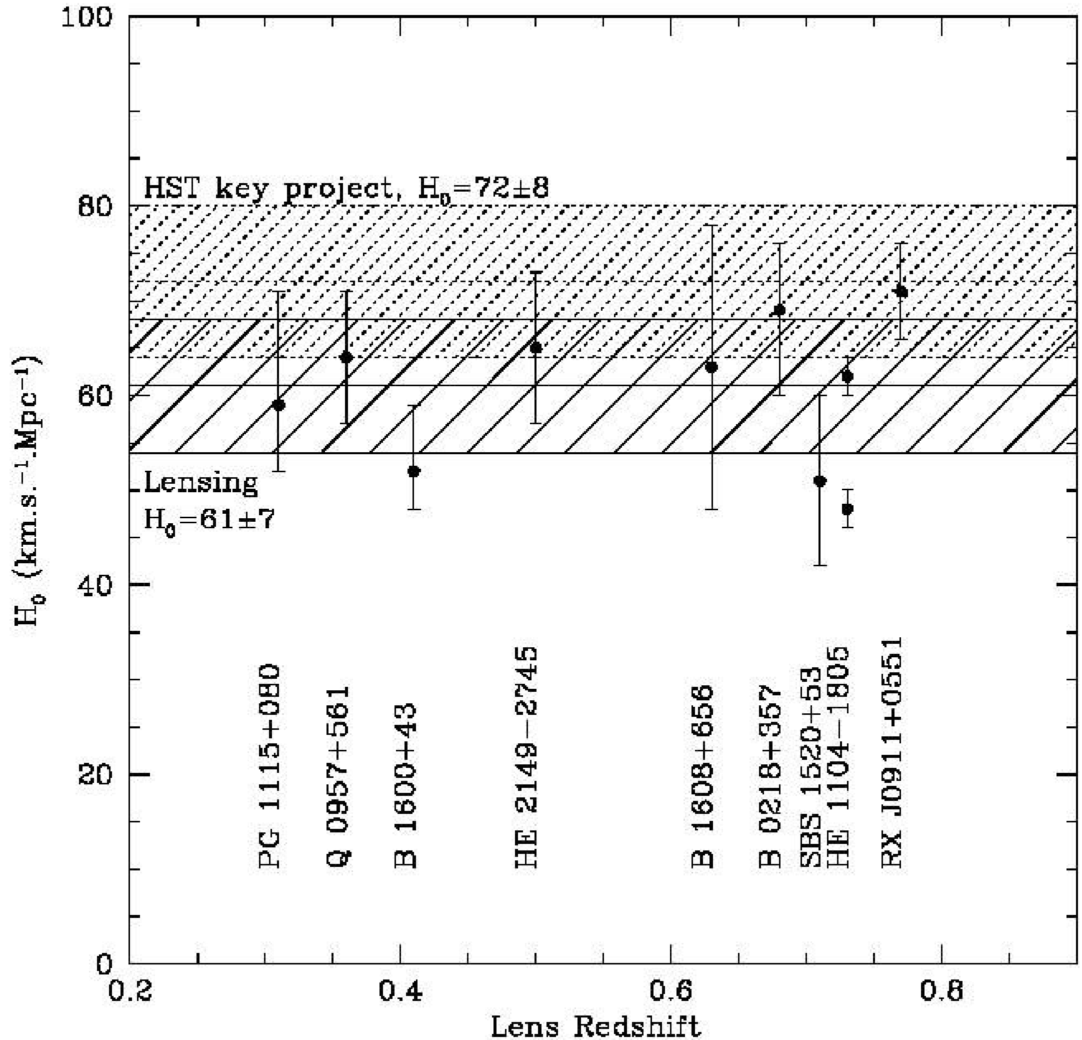


Fig. 9. Light curves of the two images of the QSO 0957+561A,B in two different filters. The two light curves have been shifted in time relative to each other by the measured time delay of 417 days, and in flux according to the flux ratio. The sharp drop measured in image A in Dec. 1994 and subsequently in image B in Feb. 1996 provides an accurate measurement of the time delay (data from Kundić et al. 1997)

Determining H_0 from lensing time delays

Table 1.1. *Time Delay Measurements*

System	N_{im}	Δt (days)
HE1104–1805	2	161 ± 7
PG1115+080	4	25 ± 2
SBS1520+530	2	130 ± 3
B1600+434	2	51 ± 2
HE2149–2745	2	103 ± 12
RXJ0911+0551	4	146 ± 4
Q0957+561	2	417 ± 3
B1608+656	4	77 ± 2
B0218+357	2	10.5 ± 0.2
PKS1830–211	2	26 ± 4
B1422+231	4	(8 ± 3)



Schechter, 2005, IAU Symposium 225, 281

Time delay of the circularly symmetric lenses

- Total time delay between two images (up to the first order):

$$\boxed{\Delta t = \tau_2 - \tau_1 = \frac{D_{\Delta t}}{c}(\theta_2^2 - \theta_1^2)\left[1 - \langle\kappa\rangle - O\left(\left(\frac{\Delta\theta}{\langle\theta\rangle}\right)^2\right)\right],}$$

(Kochanek & Schechter, 2004, astro-ph/0306040)

where $<\kappa>$ is the mean surface density in the annulus between the images, $\Delta\theta$ and $<\theta>$ are the width and average radius of the annulus

- In practice, the second order term in Δt should not be neglected
- Power law density profiles for the annulus:

$$\rho \sim r^{-\eta} \Rightarrow \langle\kappa\rangle \simeq \frac{3-\eta}{2}$$

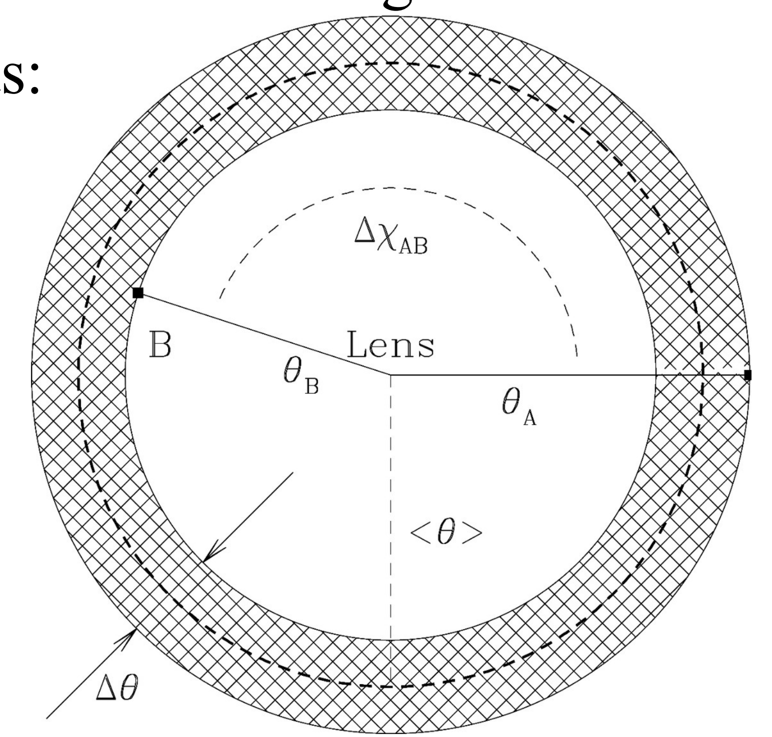
- **SIS** lens:

$$\eta = 2 \Rightarrow \langle\kappa\rangle = \frac{1}{2} \Rightarrow \Delta t_{SIS} = \frac{D_{\Delta t}}{2c}(\theta_2^2 - \theta_1^2)$$

- **Point-like** lens:

$$\eta = 3 \Rightarrow \langle\kappa\rangle = 0 \Rightarrow \Delta t_P = 2\Delta t_{SIS}$$

- Realistic H_0 estimate: between SIS and point-like case



Time-delay distance

- Expressing $D_{\Delta t}$ in terms of D_C:

$$\Omega_\kappa = 0 \Rightarrow D_A(z) = \frac{D_C(z)}{1+z} \ \wedge \ D_A(z_1, z_2) = \frac{D_C(z_2) - D_C(z_1)}{1+z_2} \Rightarrow$$

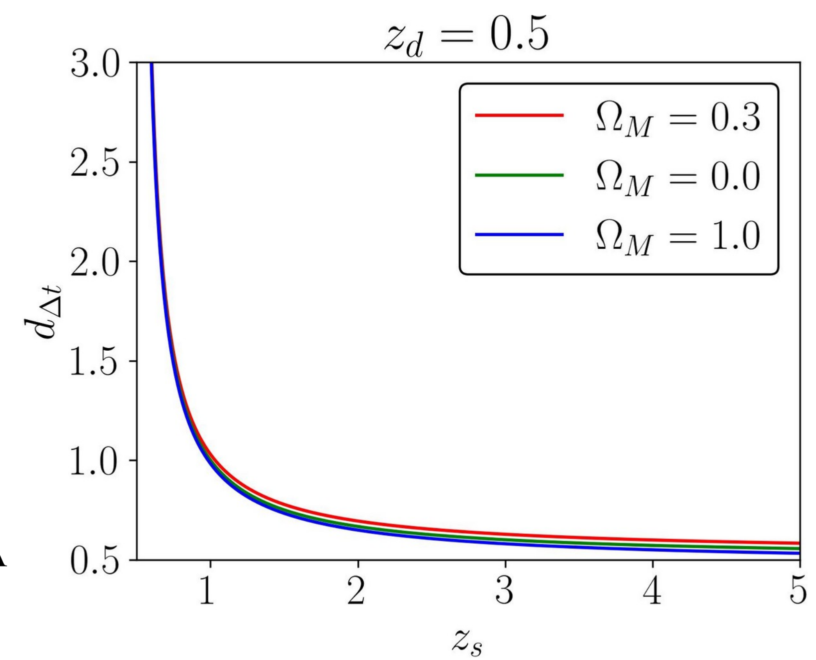
$$D_{\Delta t} = (1+z_d)\frac{D_d D_s}{D_{ds}} = \frac{D_C(z_d)\cdot D_C(z_s)}{D_C(z_s) - D_C(z_d)} = \frac{c}{H_0} d_{\Delta t} \Rightarrow$$

- Dimensionless time-delay distance:

$$d_{\Delta t} = \frac{H_0}{c}\cdot\frac{D_C(z_d)\cdot D_C(z_s)}{D_C(z_s) - D_C(z_d)} = \frac{d_C(z_d)\cdot d_C(z_s)}{d_C(z_s) - d_C(z_d)}, \quad d_C(z) = \int_0^z \frac{dz'}{\sqrt{\Omega_M(1+z')^3 + \Omega_\Lambda}}$$

- Time-delay: $\boxed{\Delta t \approx \frac{d_{\Delta t}}{H_0}(\theta_2^2 - \theta_1^2)(1 - \langle\kappa\rangle)}$

- $d_{\Delta t}$ strongly depends on z_d and z_s, very weakly on Ω_M and Ω_Λ, and **does not depend on H_0 at all**



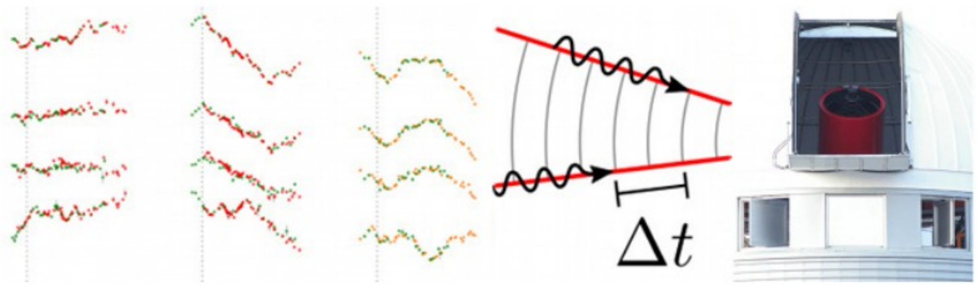
Right: dependence of $d_{\Delta t}$ on z_s, Ω_M and Ω_Λ (obtained by Python script "ddt.py")

Influence of matter structures along the line of sight

- Galaxies and other matter structures along the line of sight and close in projection to the main deflector may weakly focus and defocus the light rays from the source, and thus may influence the observed images and time delays
- The net effect of (de)focusing by such matter could be described by a constant **external convergence term $\boldsymbol{\kappa_{\mathrm{ext}}}$** in the lens model
- κ_{ext} may be positive or negative depending on whether focusing is larger than defocusing or vice versa
- Relation between the corrected ($D_{\Delta t}$) and the effective ($D_{\Delta t}^{\mathrm{model}}$) time-delay distance predicted by the model: $$\boxed{D_{\Delta t} = \frac{D_{\Delta t}^{\mathrm{model}}}{1-\kappa_{\mathrm{ext}}}}$$
- The corresponding relation for the corrected H_0: $$\boxed{H_0 = (1-\kappa_{\mathrm{ext}}) \cdot H_0^{\mathrm{model}}}$$

where H_0^{model} is obtained by neglecting the weak external perturbers

COSMOLOGICAL MONITORING OF GRAVITATIONAL LENSES COSMOGRAIL



In short

COSMOGRAIL is a decade-long monitoring program aimed at measuring the time delays between the multiple images of most known lensed quasars, primarily in order to constrain the Hubble constant.

Welcome

COSMOGRAIL is the **COS**mological **MO**nitoring of **GRA**v**I**tational **L**enses. Our project is aimed at measuring time delays for most known lensed quasars, from optical light curves obtained with small but (almost) dedicated telescopes in the northern and southern hemispheres. COSMOGRAIL involves people from different countries, spanning a broad range of expertises. The goal is to measure individual time delays with an accuracy below 3%, in order to determine the Hubble constant H_0. A standalone measurement of H_0 is highly complementary to other cosmological probes, such as the observations of the CMB made by the ESA Planck mission. This complementarity will help us to understand better the so called "dark energy" which is driving the accelerated expansion of our Universe.

Measuring time delays is difficult, but not as difficult as it first appeared in the late 80s when the first monitoring programs were started. Obtaining regular observing time on telescopes in good sites was (and is still) not easy and the small angular separations between the quasar images require to perform accurate photometry of blended objects, sometimes with several quasar images plus the lensing galaxy within the seeing disk. The COSMOGRAIL project, started in April 2004 and lead by EPFL, addresses both issues of carrying out photometry of faint blended sources and of obtaining well sampled light curves. In order to use all the available data, even with bad seeing, the data are processed using the MCS deconvolution algorithm.

A presentation of COSMOGRAIL and its scientific motivation can be found in our recent ESO Messenger article.

CONTACT

For questions or remarks about COSMOGRAIL in general, please contact the project PIs Georges Meylan and Frédéric Courbin.

When referring to this website in a publication, please use the URL www.cosmograil.org.

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University of Belgrade
Faculty of Mathematics

Erasmus Mundus Joint European Master Studies in Astrophysics - AstroMundus

astro mundus international master

Master Thesis

The Hubble constant from time-delays of gravitationally lensed quasars

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Belgrade Serbia September 2017

Source	z_s	z_l	Δt_{SIS}	Δt_{SIE}	Δt_{DV}
JVAS B0218+357	0.944	0.6847	4.9	4.8	6.0
SBS 0909+532	1.377	0.830	29.2	27.2	9.2
RX J0911.4+0551 (A-B)	2.8	0.769	–	148.9	213.2
RX J0911.4+0551 (A-C)	2.8	0.769	–	147.9	211.9
RX J0911.4+0551 (A-D)	2.8	0.769	–	149.5	214.2
FBQS J0951+2635	1.246	0.260	13.8	16.5	20.8
HE 1104-1805	2.319	0.729	185.2	189.9	268.9
PG 1115+080 (A-B)	1.722	0.3098	–	10.4	15.7
PG 1115+080 (A-C)	1.722	0.3098	–	10.5	15.9
PG 1115+080 (A-D)	1.722	0.3098	–	17.0	26.1
JVAS B1422+23 (A-B)	3.62	0.3366	–	0.7	1.3
JVAS B1422+231 (A-C)	3.62	0.3366	–	0.9	1.7
JVAS B1422+231 (A-D)	3.62	0.3366	–	24.6	39.7
SBS 1520+530	1.855	0.761	93.6	95.9	122.3
CLASS B1600+434	1.59	0.41	27.6	39.5	44.6
CLASS B1608+656 (A-B)	1.394	0.6304	–	48.1	27.7
CLASS B1608+656 (A-C)	1.394	0.6304	–	53.9	32.1
CLASS B1608+656 (A-D)	1.394	0.6304	–	115.8	102.0
HE 2149-2745	2.033	0.603	93.7	88.6	116.8

Table 5.1: Hubble constant estimates for the SIS model in [km/s/Mpc].

Quasar	$H_0(cosmology1)$	$H_0(cosmology2)$
JVAS B0218+357	74.75	77.46
SBS 0909+532	75.27	79.08
FBQS J0951+2635	72.56	73.96
HE 1104-1805	76.89	81.22
SBS 1520+530	74.65	78.41
CLASS B1600+434	79.29	81.83
HE 2149-2745	66.18	69.33

Table 5.2: Averaged values and standard deviation

SIS Model	H_0
Cosmology 1	74.49 ± 4.21
Cosmology 2	77.33 ± 4.38

Exam question

1. Fermat potential, lensing time delay and its application for determination of H_0

Literature

Textbook:

- *Gravitational Lensing*: Strong, Weak and Micro, Book Series: Saas-Fee Advanced Courses
  1. P. Schneider - *Introduction to Gravitational Lensing and Cosmology*
  2. C. S. Kochanek - *Strong Gravitational Lensing*
  3. P. Schneider - *Weak Gravitational Lensing*
  4. J. Wambsganss - *Gravitational Microlensing*

Exercise 1

Measured time delay between 2 images of gravitational lens system HE2149-2745 is 103 days, positions of the images in respect to the lens galaxy are: x_A = 0".714, y_A = -1".150, and x_B = -0".176, y_B = 0".296, and redshifts of the source and lens are: z_s = 2.03 and z_d = 0.50. Estimate the value of H_0, assuming SIS and point-like lens models, as well as the following cosmological model: $\Omega_M = 0$ and $\Omega_\Lambda = 1$.

Exercise 2

Estimate the value of H_0 from HE2149-2745 time delay (take all necessary data from Exercise 1), assuming $<\kappa> = 0.22$ (Kochanek, 2002, ApJ, 578, 25), and the realistic cosmological model: $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$. Use the Ned Wright's Javascript Cosmology Calculator (http://www.astro.ucla.edu/~wright/CosmoCalc.html) to quickly calculate the dimensionless time-delay distance.

Solution 1

- SIS lens:

$$\Omega_M = 0 \wedge \Omega_\Lambda = 1 \Rightarrow d_C(z) = \int_0^z \frac{dz'}{\sqrt{\Omega_M(1+z')^3 + \Omega_\Lambda}} = z \Rightarrow$$

$$d_{\Delta t} = \frac{d_C(z_d) \cdot d_C(z_s)}{d_C(z_s) - d_C(z_d)} = \frac{z_d z_s}{z_s - z_d}$$

$$\Delta t_{SIS} = \frac{D_{\Delta t}}{2c}(\theta_A^2 - \theta_B^2) = \frac{d_{\Delta t}}{2H_0}(\theta_A^2 - \theta_B^2) \Rightarrow H_0 = \frac{z_d z_s (\theta_A^2 - \theta_B^2)}{2(z_s - z_d)\Delta t_{SIS}}$$

$$\left.\begin{array}{l} \theta_A = 1''.354 = 6.564 \times 10^{-6}\mathrm{rad} \\ \theta_B = 0''.344 = 1.668 \times 10^{-6}\mathrm{rad} \end{array}\right\} \Rightarrow \theta_A^2 - \theta_B^2 = 4.03 \times 10^{-11}$$

$$H_0 = \frac{2.03 \cdot 0.50 \cdot 4.03 \times 10^{-11}}{2(2.03 - 0.50) \cdot 103 \cdot 86400\,\mathrm{s}} \cdot \frac{3.0856776 \times 10^{19}\,\mathrm{km}}{\mathrm{Mpc}} = 46.35\,\frac{\mathrm{km}}{\mathrm{s} \cdot \mathrm{Mpc}}$$

- Point-like lens:

$$\Delta t_P = 2\Delta t_{SIS} \Rightarrow H_0 = 2 \cdot 46.35\,\frac{\mathrm{km}}{\mathrm{s} \cdot \mathrm{Mpc}} = 92.7\,\frac{\mathrm{km}}{\mathrm{s} \cdot \mathrm{Mpc}}$$

Solution 2

- Dimensionless time-delay distance: $d_{\Delta t} = \frac{H_0}{c} \cdot \frac{D_C(z_d) \cdot D_C(z_s)}{D_C(z_s) - D_C(z_d)}$
- Hint: we can use an arbitrary value of H_0 to calculate the required comoving distances because it will vanish by reducing the fraction for $d_{\Delta t}$, and thus it will have no effect on the final result

  For that purpose we can use e.g. H_0 = 30 km/s/Mpc:

$$H_0 = 30\,\frac{\mathrm{km}}{\mathrm{s}\cdot\mathrm{Mpc}} \overset{\Omega_M=0.3}{\underset{\Omega_\Lambda=0.7}{\Rightarrow}} d_{\Delta t} = \frac{30}{300000} \cdot \frac{4405.7 \cdot 12175.8}{12175.8 - 4405.7} = 0.69$$

$$H_0 = \frac{d_{\Delta t}}{\Delta t}(\theta_2^2 - \theta_1^2)(1 - \langle\kappa\rangle)$$

$$H_0 = \frac{0.69 \cdot 4.03 \times 10^{-11} \cdot (1 - 0.22)}{103 \cdot 86400\,\mathrm{s}} \cdot \frac{3.0856776 \times 10^{19}\,\mathrm{km}}{\mathrm{Mpc}} \Leftrightarrow H_0 = 75.2\,\frac{\mathrm{km}}{\mathrm{s}\cdot\mathrm{Mpc}}$$

- Note: the above result could be further improved by taking into account the second order term in Δt