MASS 2023 Course:

Gravitational Lenses

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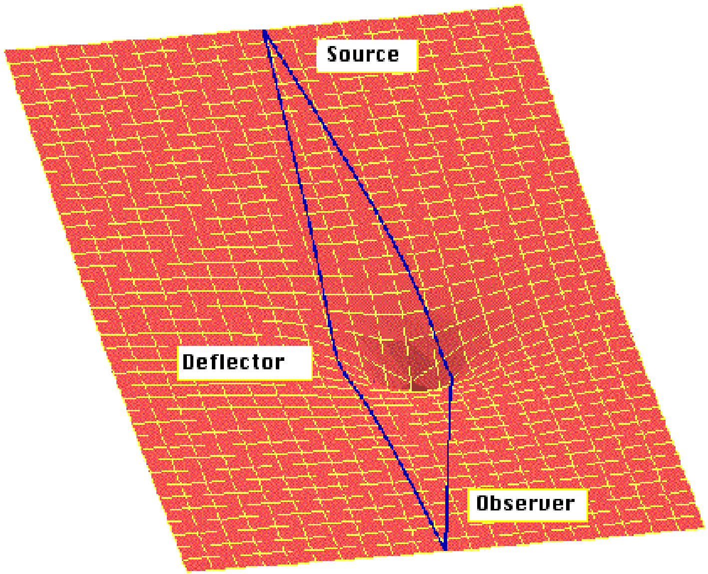
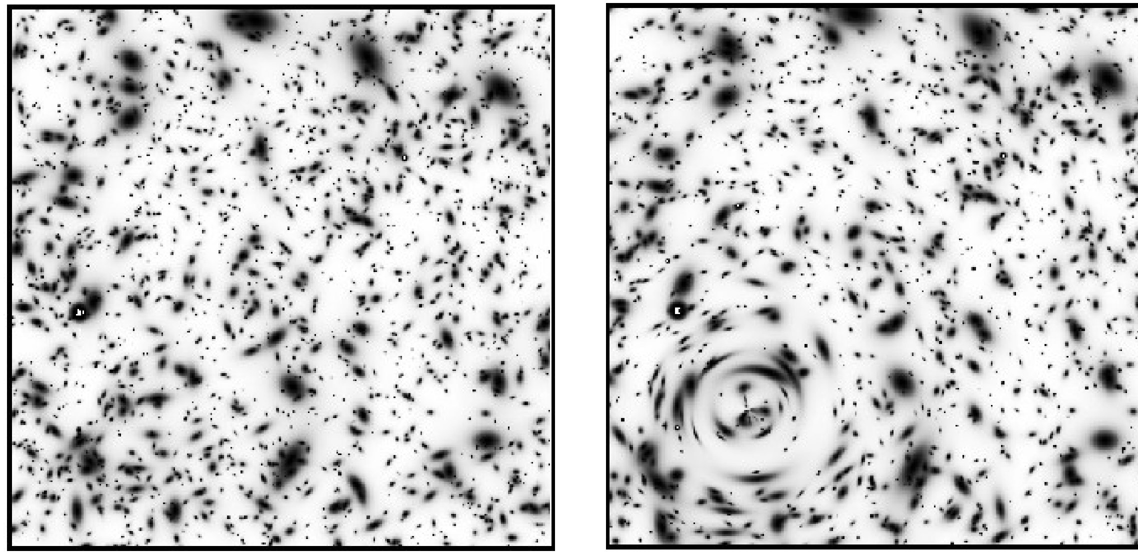
Lecture 10

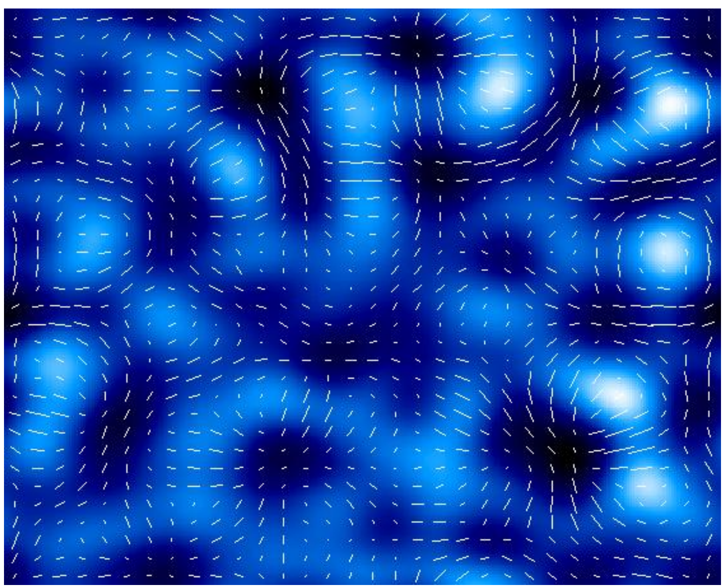
1. **Weak gravitational lensing**
   - Basic principles
   - Shape distortions due to weak lensing: reduced shear
   - Mass reconstructions from weak lensing
   - Dark matter and its alternatives
   - Detection of dark matter by weak lensing
2. Exercise

Basic principles of weak lensing

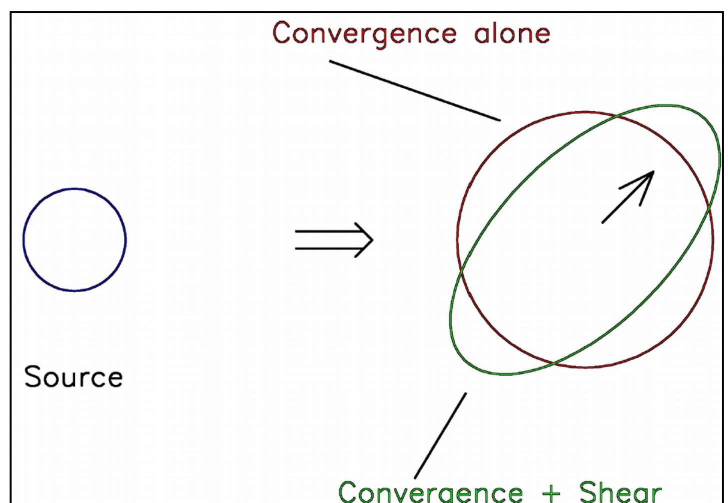


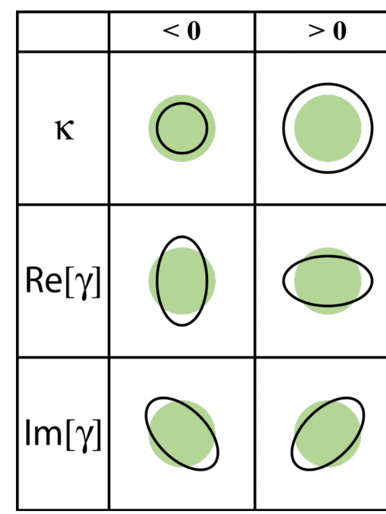


Background galaxies in absence of lensing (left), and in the case of weak lensing by invisible foreground cluster of galaxies (right)



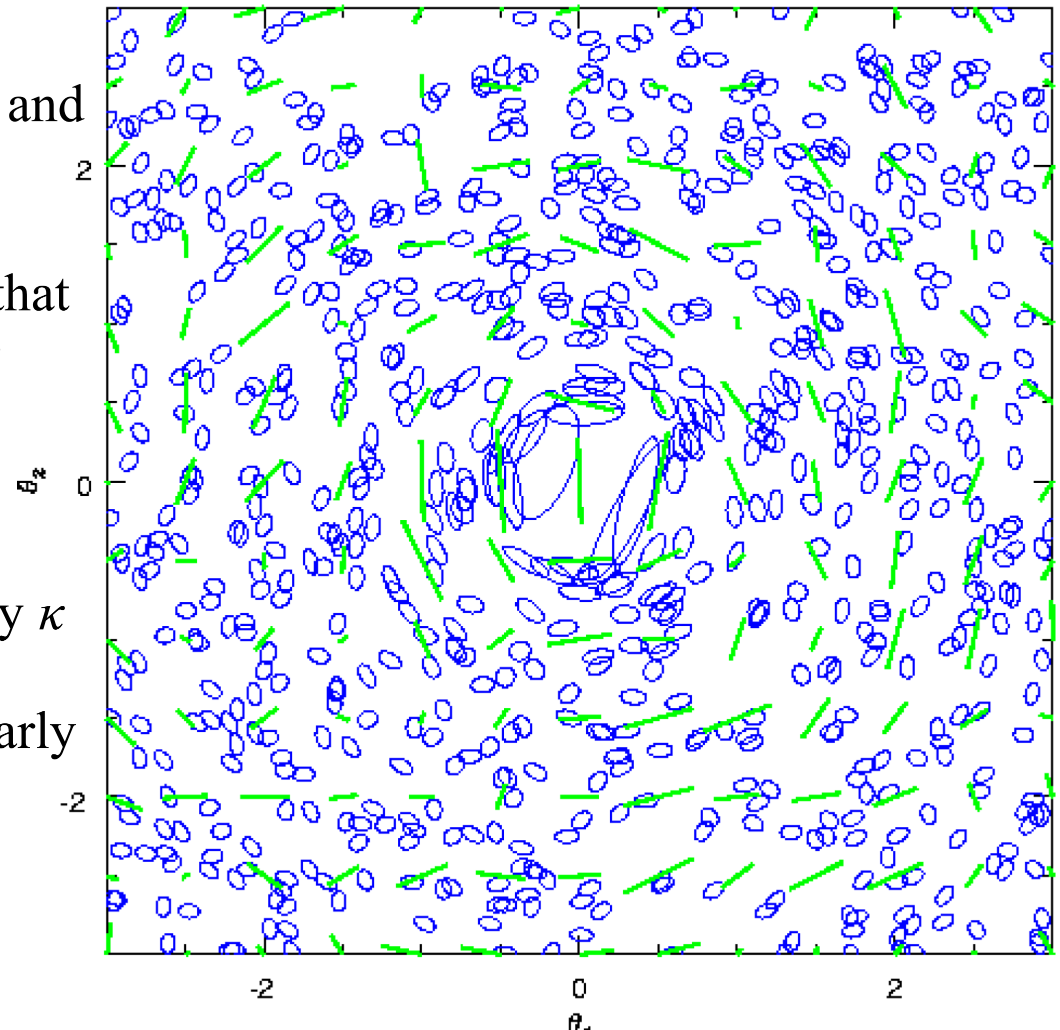
Projected mass (shades of blue) and shear (white sticks)





Weak lensing: shear and surface mass density

- An observed galaxy ellipticity is a combination of its intrinsic ellipticity and shear
- Shear can be estimated by averaging over many galaxy images, assuming that the intrinsic ellipticities are *randomly oriented*
- Thus, weak lensing can be used for measuring the local shear γ
- Both, shear γ and surface mass density κ are second partial derivatives of the deflection potential $\psi \Rightarrow$ they are linearly related
- Therefore, it is possible to derive the surface mass density κ from the measured values of shear γ

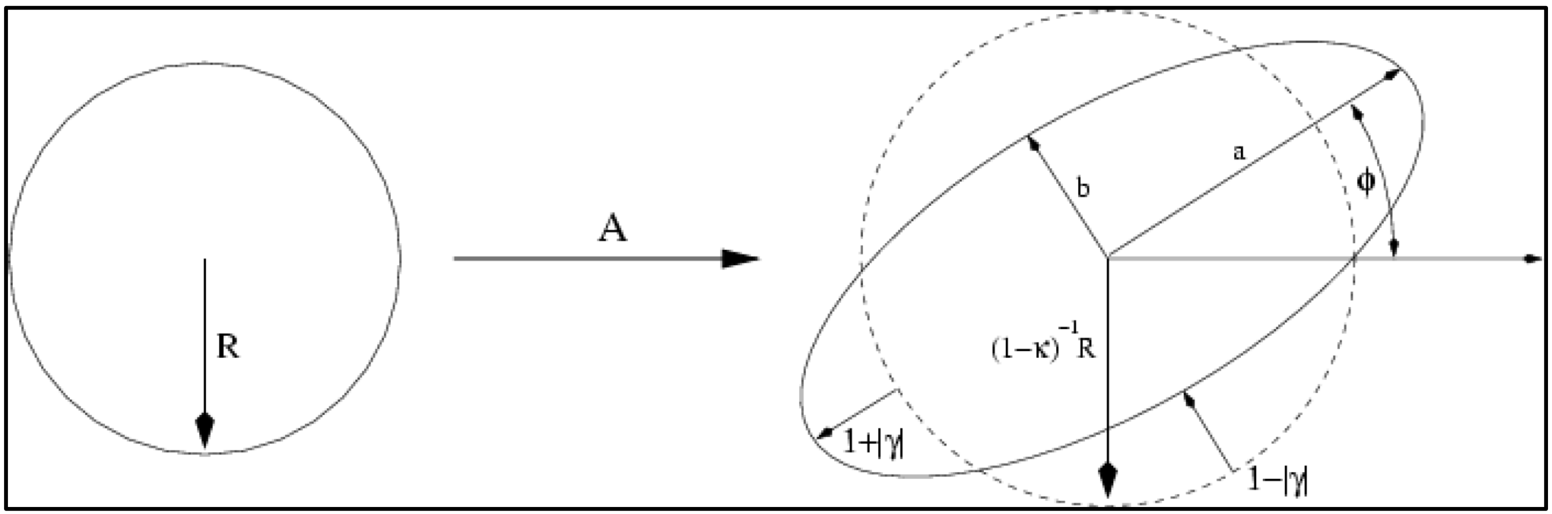


- This technique is called **mass reconstruction** and is usually used in the case of clusters of galaxies
- Local averages of ellipticities of background galaxies $\Rightarrow$ local shear estimates (green sticks) $\Rightarrow$ surface mass density mass map

Shape distortions due to weak lensing

- distortion matrix: $A(\vec{\theta}) = (1-\kappa)\begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix}$ where $g(\vec{\theta}) = \dfrac{\gamma(\vec{\theta})}{[1-\kappa(\vec{\theta})]}$ is **reduced shear**
- mapping of a circular source of radius *R* to an ellipse with axes:

$$\frac{R}{1-\kappa-|\gamma|} = \frac{R}{(1-\kappa)(1-|g|)}; \quad \frac{R}{1-\kappa+|\gamma|} = \frac{R}{(1-\kappa)(1+|g|)}$$



- the measured image ellipticities of a circular source yield the value of the reduced shear through the axis ratio: $|g| = \dfrac{1-b/a}{1+b/a} \Leftrightarrow \dfrac{b}{a} = \dfrac{1-|g|}{1+|g|}$
- not the shear γ, but only the reduced shear g can be determined from galaxy ellipticities

Mass reconstructions from weak lensing

1. The Kaiser–Squires inversion:

- relation between shear γ and convergence κ in form of a convolution with the kernel D which describes the shear generated by a point mass:

$$\gamma(\vec{\theta}) = \frac{1}{\pi}\int_{R^2} d^2\theta' \, D(\vec{\theta}-\vec{\theta}')\kappa(\vec{\theta}'), \quad D(\vec{\theta}) \equiv -\frac{\theta_1^2-\theta_2^2+2i\theta_1\theta_2}{|\vec{\theta}|^4} = \frac{-1}{(\theta_1 - i\theta_2)^2}$$

- in Fourier space this convolution becomes a multiplication:

$\hat{\gamma}(\vec{l}) = \pi^{-1}\hat{D}(\vec{l})\hat{\kappa}(\vec{l}) \quad \text{for} \quad \vec{l} \neq 0$ which can be inverted to yield:

$\hat{\kappa}(\vec{l}) = \pi^{-1}\hat{\gamma}(\vec{l})\hat{D}^*(\vec{l}) \quad \text{for} \quad \vec{l} \neq 0, \text{ where } D \cdot \hat{D}^* = \pi^2$

- Fourier back-transformation and by taking its real part ⇒ $$\kappa(\vec{\theta}) - \kappa_0 = \frac{1}{\pi}\int_{\mathbb{R}^2} d^2\theta' \mathcal{R}\left[D^*(\vec{\theta}-\vec{\theta}')\gamma(\vec{\theta}')\right]$$
- Problem: integral extends over $\mathbb{R}^2$ but data are available only on a finite field

2. Finite-field inversions:

- κ is obtained from a shear measurement on a finite field
- a *local* relation γ and κ (Schneider 1995, A&A, 302, 639): $\nabla\kappa = \begin{pmatrix}\gamma_{1,1}+\gamma_{2,2} \\ \gamma_{2,1}-\gamma_{1,2}\end{pmatrix}$
- in the case of reduced shear g: $$\nabla \ln(1-\kappa) = \frac{1}{1-g_1^2-g_2^2}\begin{pmatrix}1+g_1 & g_2 \\ -g_2 & 1-g_1\end{pmatrix}\begin{pmatrix}g_{1,1}+g_{2,2} \\ g_{2,1}-g_{1,2}\end{pmatrix}$$

Mass reconstruction: Example I

- Luppino & Kaiser 1997, ApJ, 475, 20:

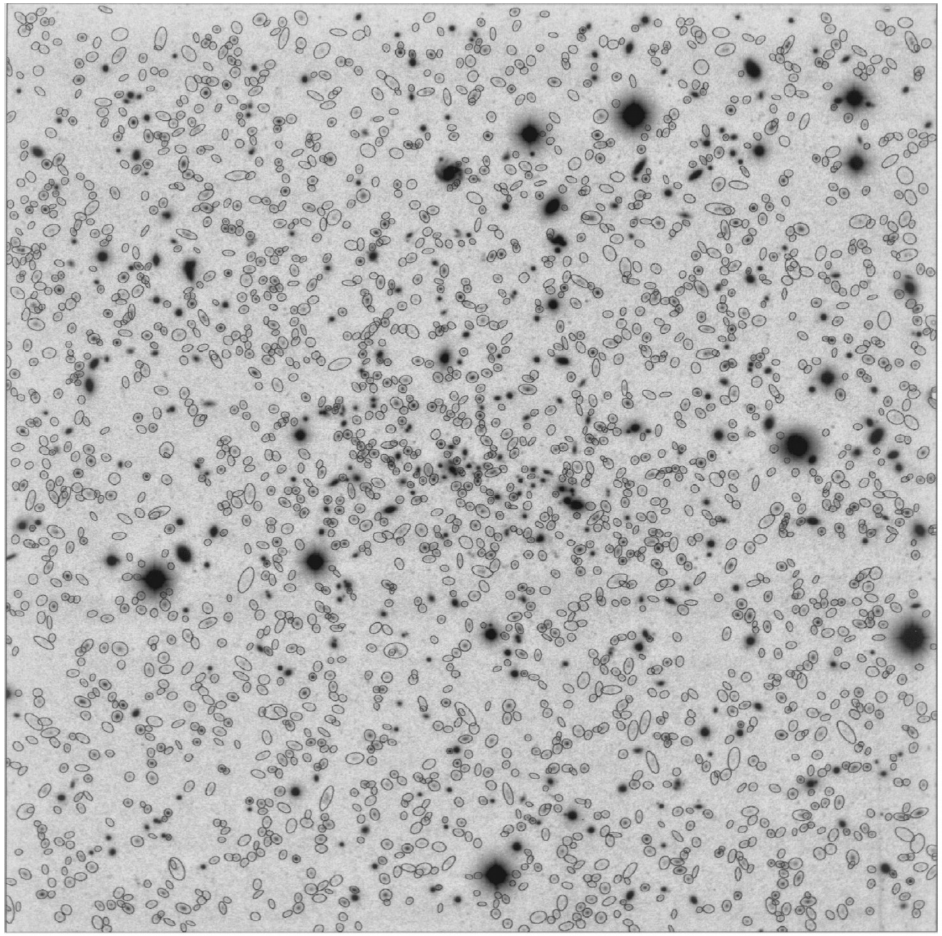


FIG. 3.—Full 2048 × 2048 pixel I-band CCD image of MS 1054−03 with the ellipses drawn around all the 2395 objects in the $I > 21.5$ catalog

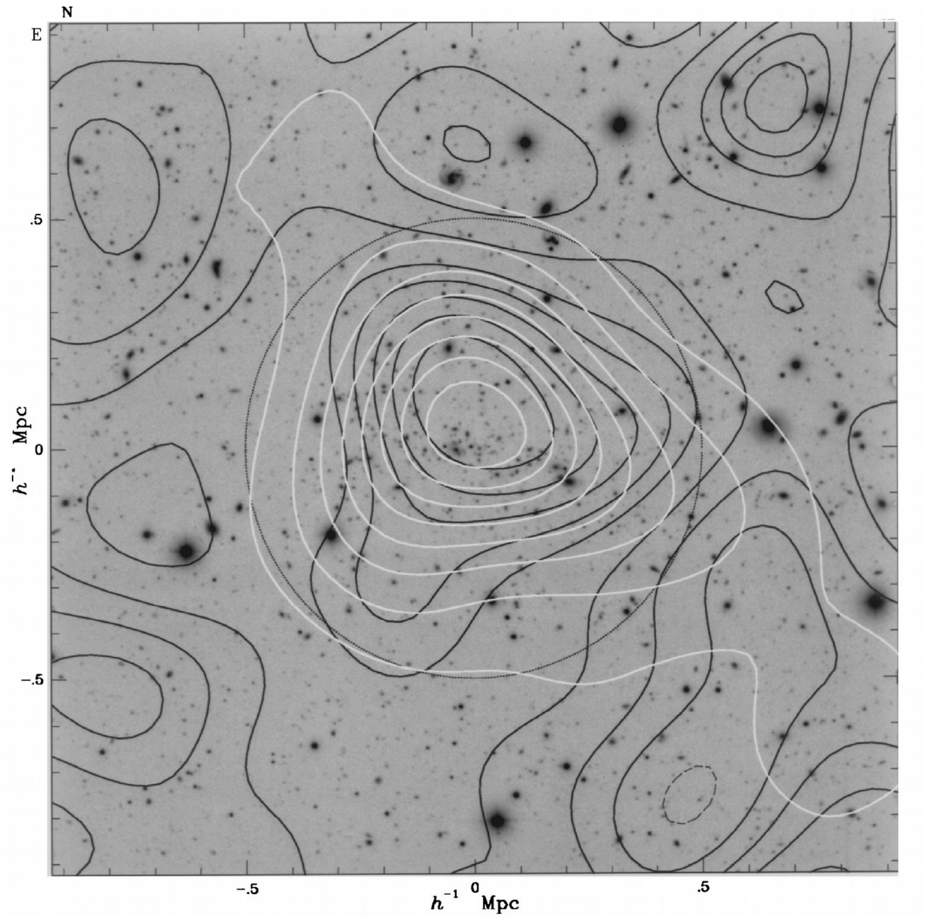
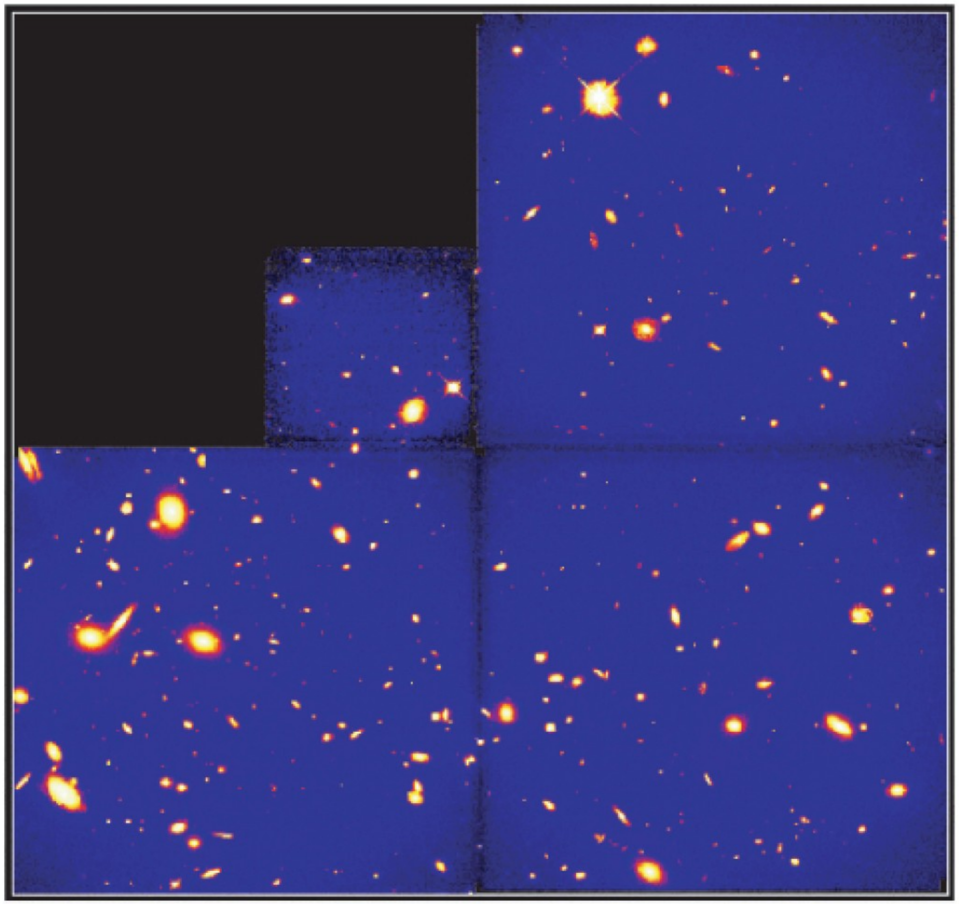


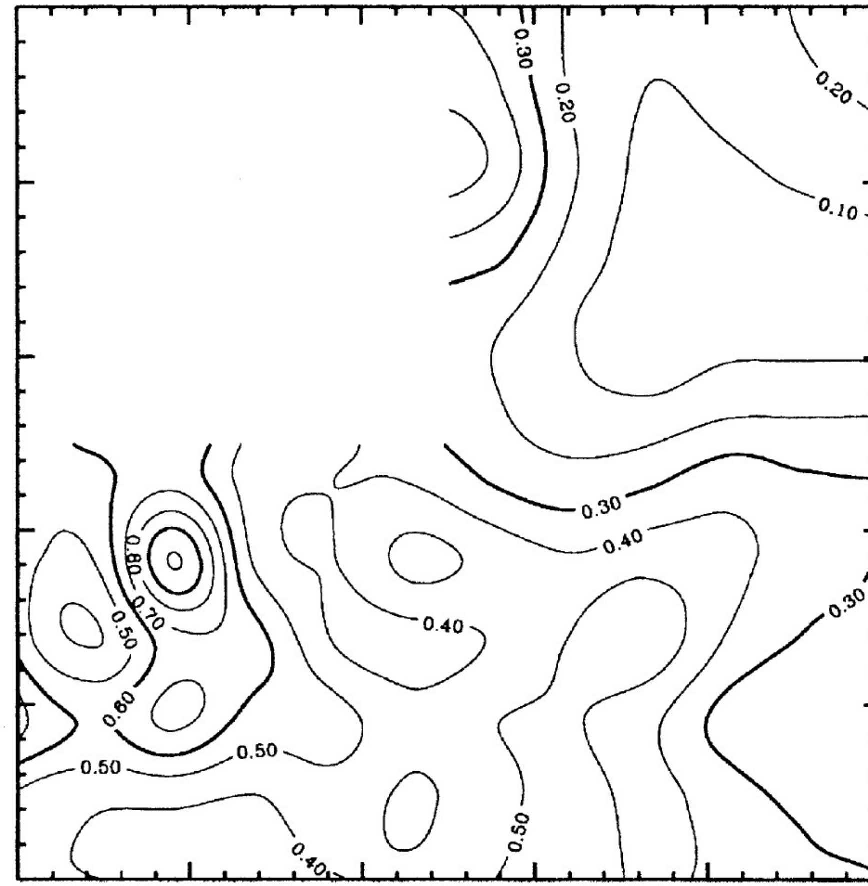
FIG. 5.—Contour plot of the surface mass density (*black contour lines*) and cluster light distribution (*white contour lines*) overlaid on the 2048^2 pixel optical image of the cluster.

Left: image of cluster of galaxies MS1054−03 with about 2400 measured galaxy ellipticities

Right: mass reconstruction (black), compared to light distribution (white)

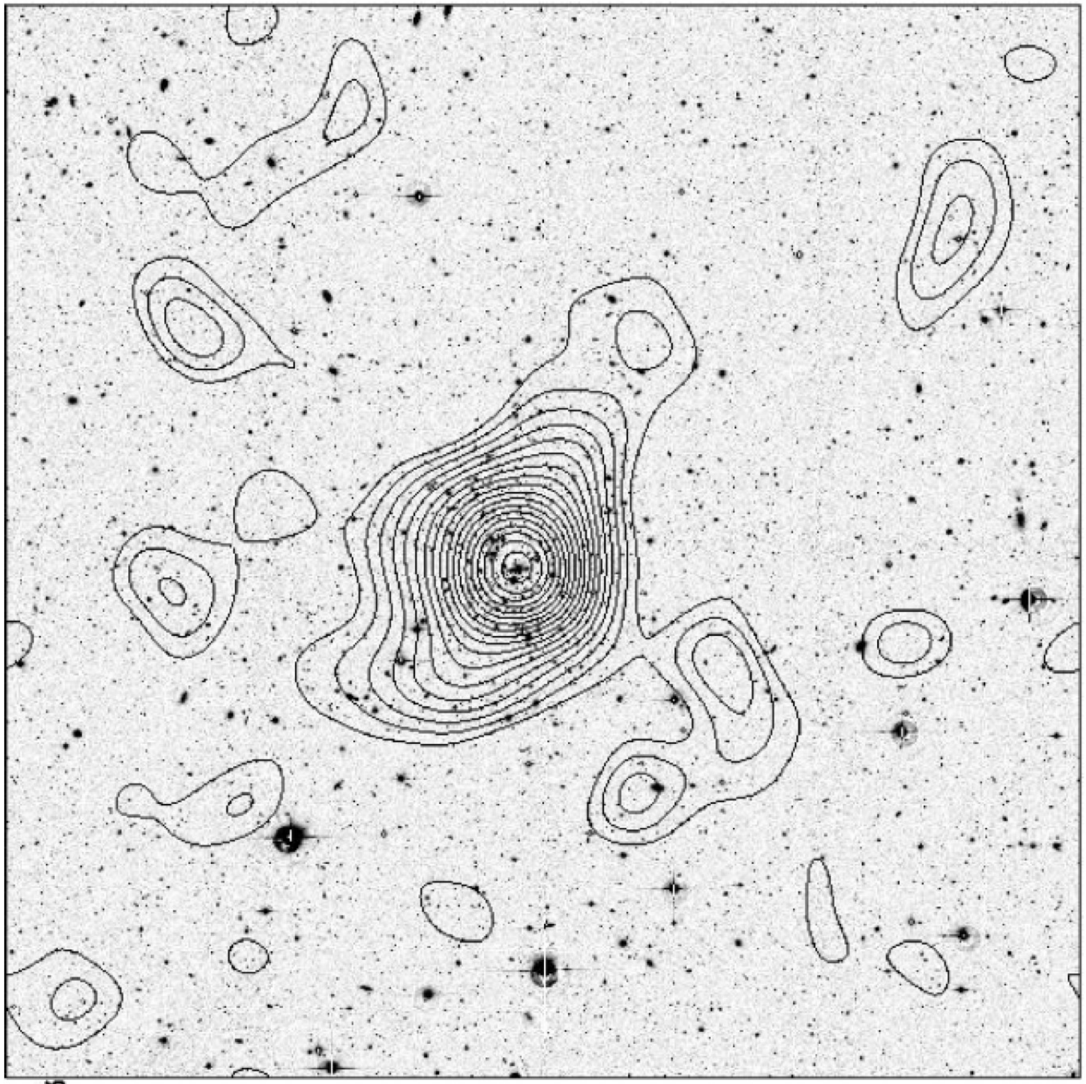
Mass reconstruction: Example II

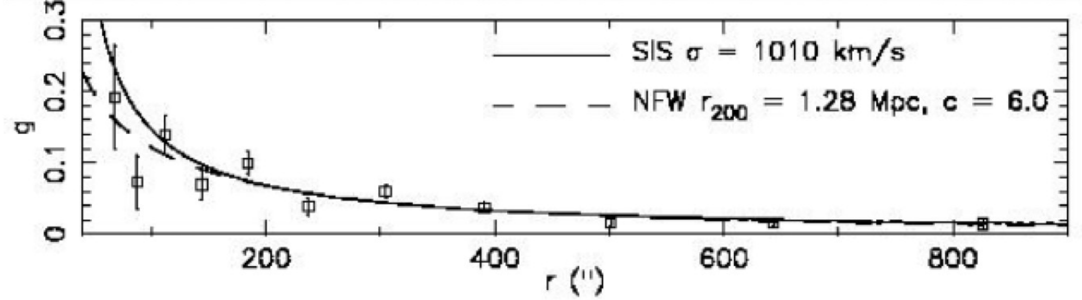




Comparison between a HST image of the cluster Abell 851 (left) and the corresponding mass reconstruction by weak lensing (right)

Mass reconstruction: Example III

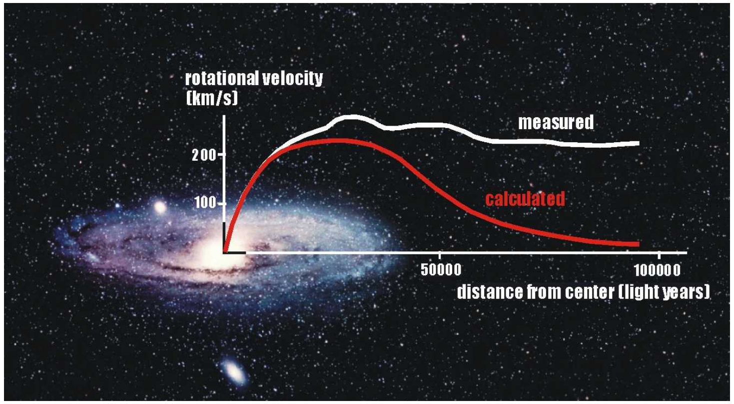




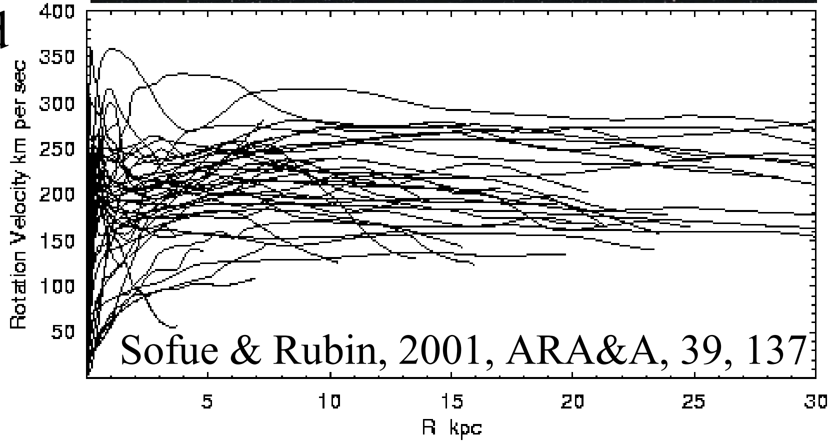
Top: contours show the mass reconstruction of the cluster A 1689 with contour spacing *$\Delta\kappa = 0.01$*

Bottom: the reduced shear profile and the best fitting SIS and NFW models

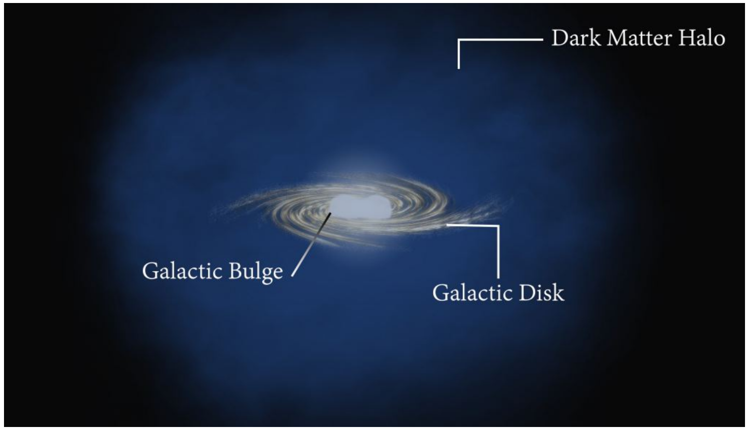
Dark matter



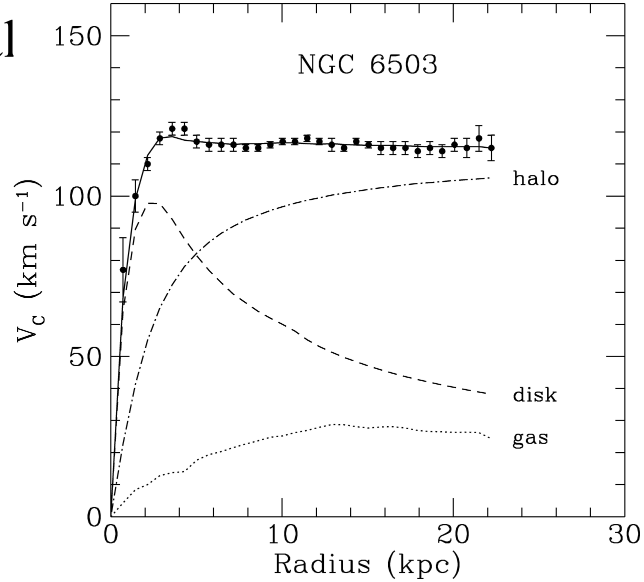
- Zwicky applied virial theorem to the motions of galaxies in the Coma Cluster ⇒ several hundred times more estimated than observable mass ⇒ "dunkle Materie" (Zwicky, 1933, HPA, 6, 110)



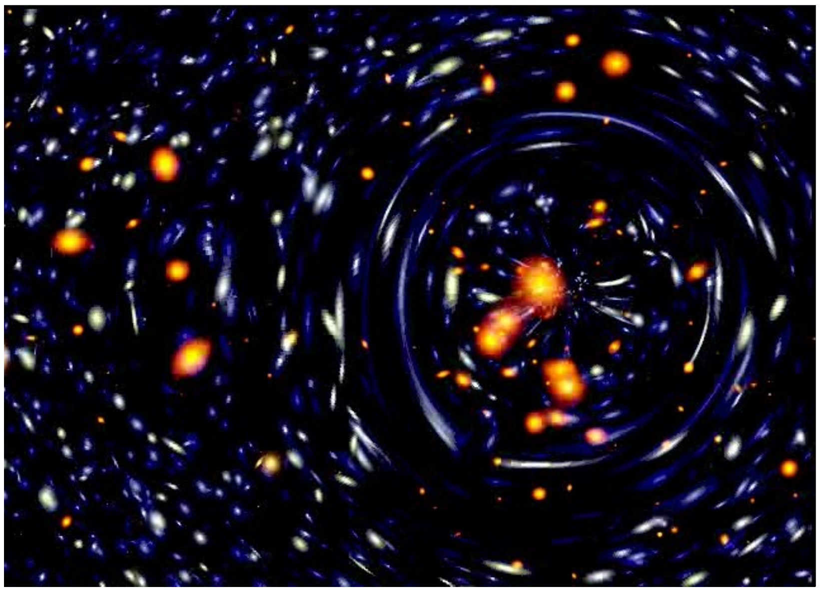
- Vera Rubin in the 1960s and 1970s: the observed rotation curves of spiral galaxies are flat ⇒ 6 times as much dark as visible mass
- DM is composed from non-baryonic particles which are so weakly interacting that they move purely under the influence of gravity ⇒ it can be directly detected only by weak lensing
- <u>Hypothesis</u>: a spherical **dark matter halo** around a spiral galaxy (Navarro, Frenk & White, 1996, ApJ, 462, 563):

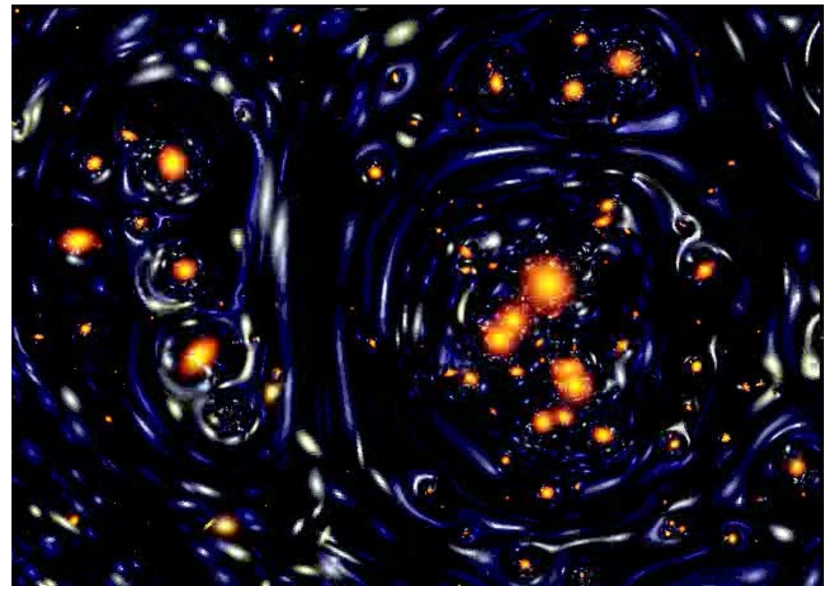


$$\rho(r)=\frac{\rho_0}{\frac{r}{r_0}\left(1+\frac{r}{r_0}\right)^2}$$



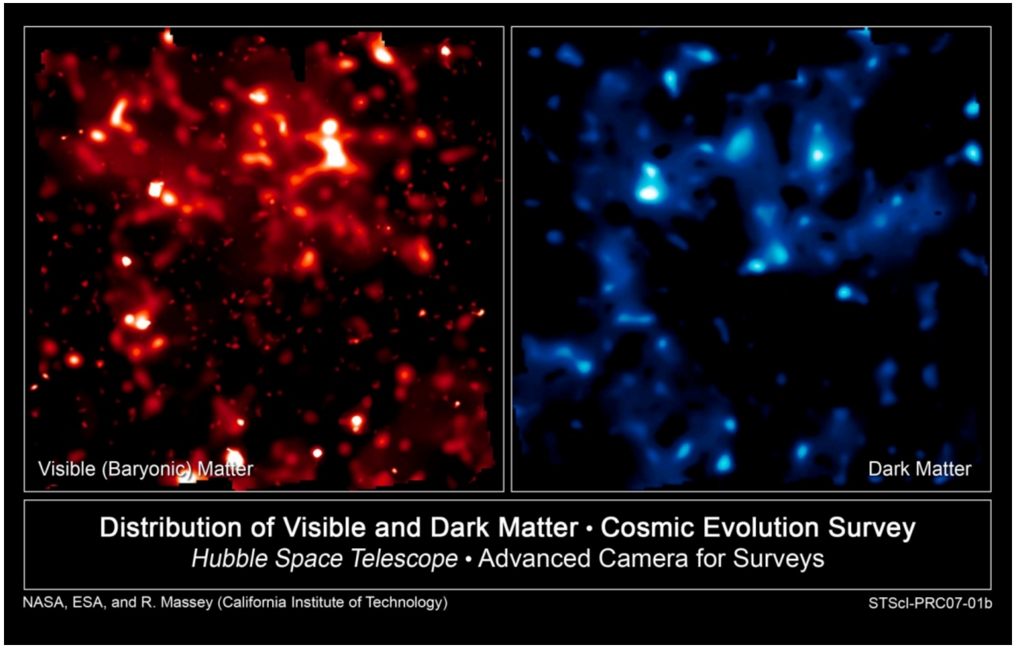
Gravitational lensing by dark matter



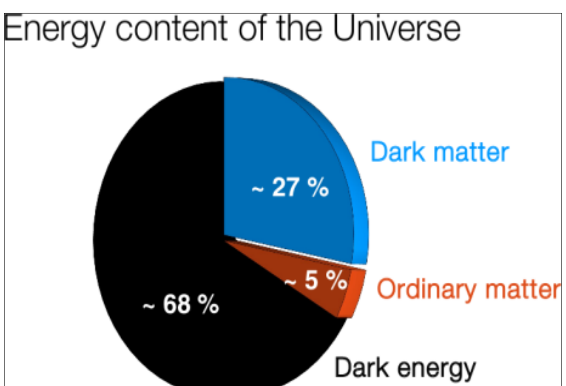


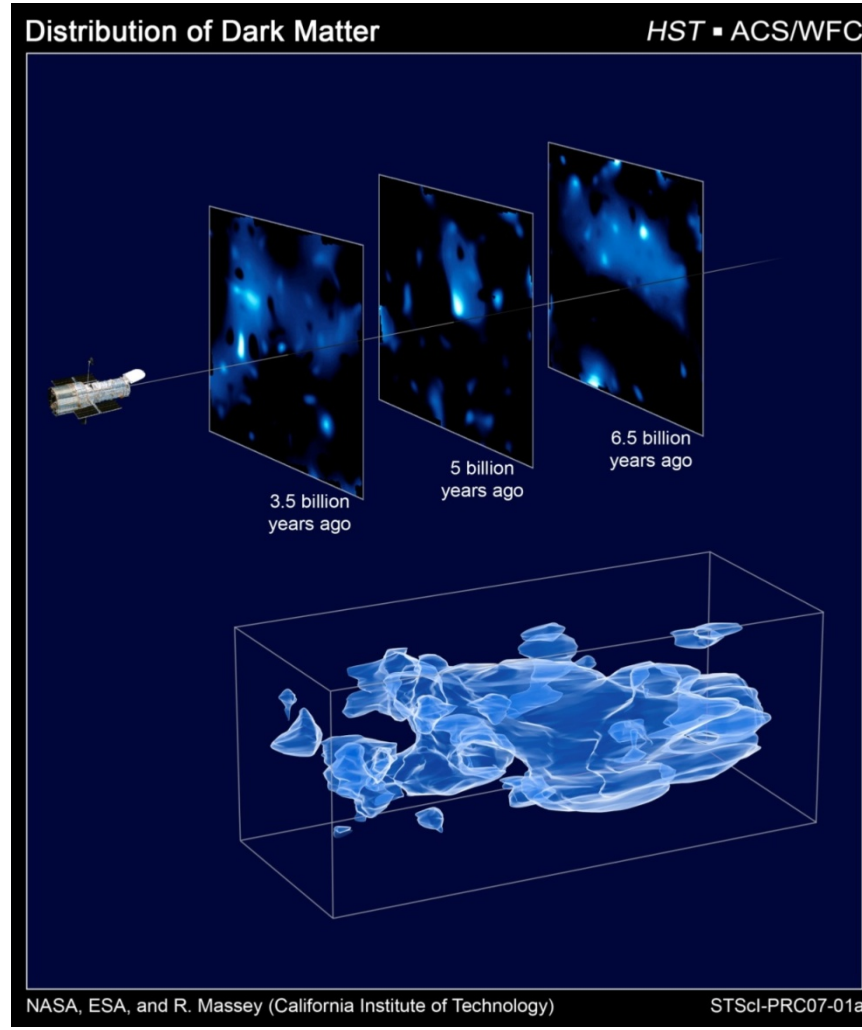
Gravitational lensing by a massive cluster of galaxies with two different distributions of the same amount of the dark matter over the cluster (orange), causing a particular distortion of the background galaxies (white and blue).

Spatial distribution of dark matter



- Dark matter constitutes about 5/6 of the total mass of the Universe





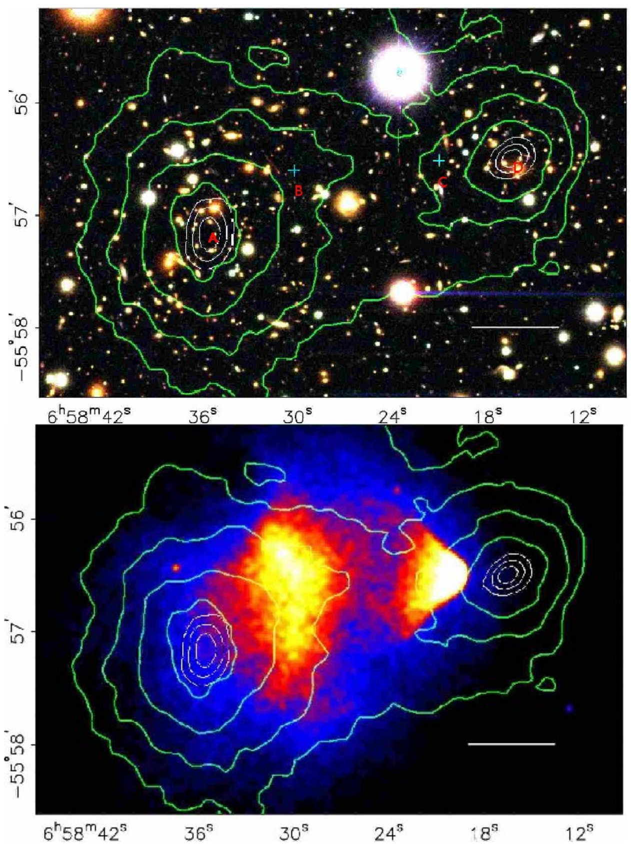
Detection of dark matter in the case of "Bullet Cluster" (1E 0657-558)

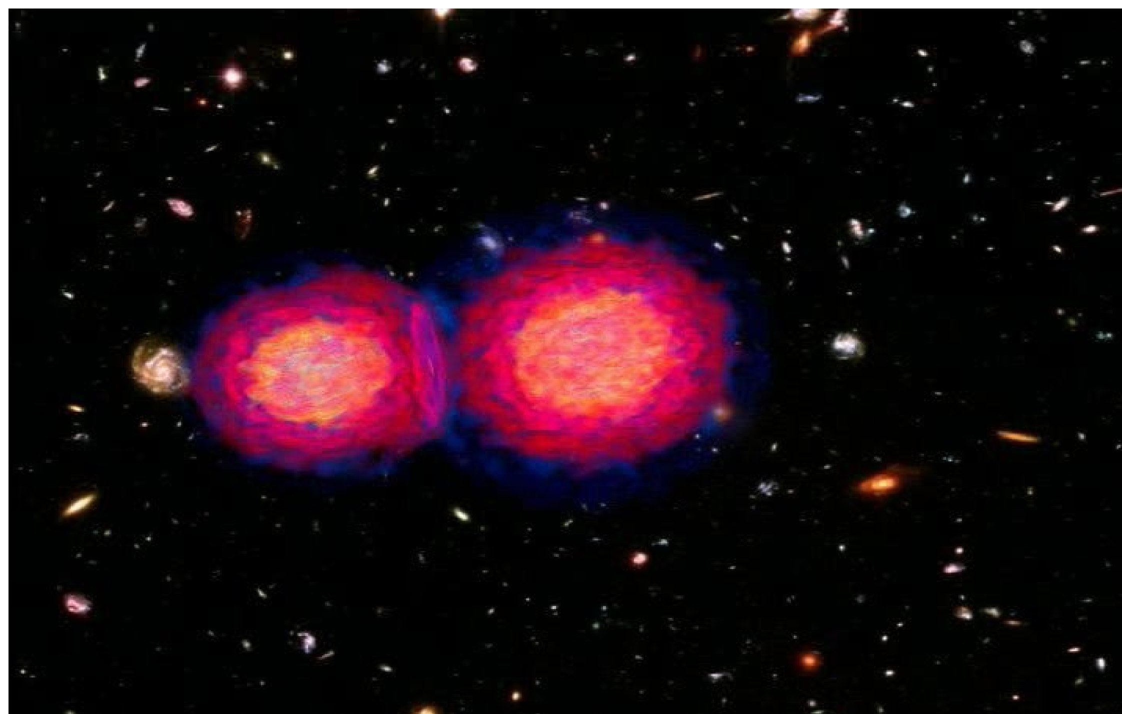
The Astrophysical Journal, 648:L109–L113, 2006 September 10

A DIRECT EMPIRICAL PROOF OF THE EXISTENCE OF DARK MATTER[1]

Douglas Clowe,[2] Maruša Bradač,[3] Anthony H. Gonzalez,[4] Maxim Markevitch,[5,6]
Scott W. Randall,[5] Christine Jones,[5] and Dennis Zaritsky[2]

Received 2006 June 6; accepted 2006 August 3; published 2006 August 30





Oposite example: Abell 520

THE ASTROPHYSICAL JOURNAL, 668:806–814, 2007 October 20

A DARK CORE IN ABELL 520[1]

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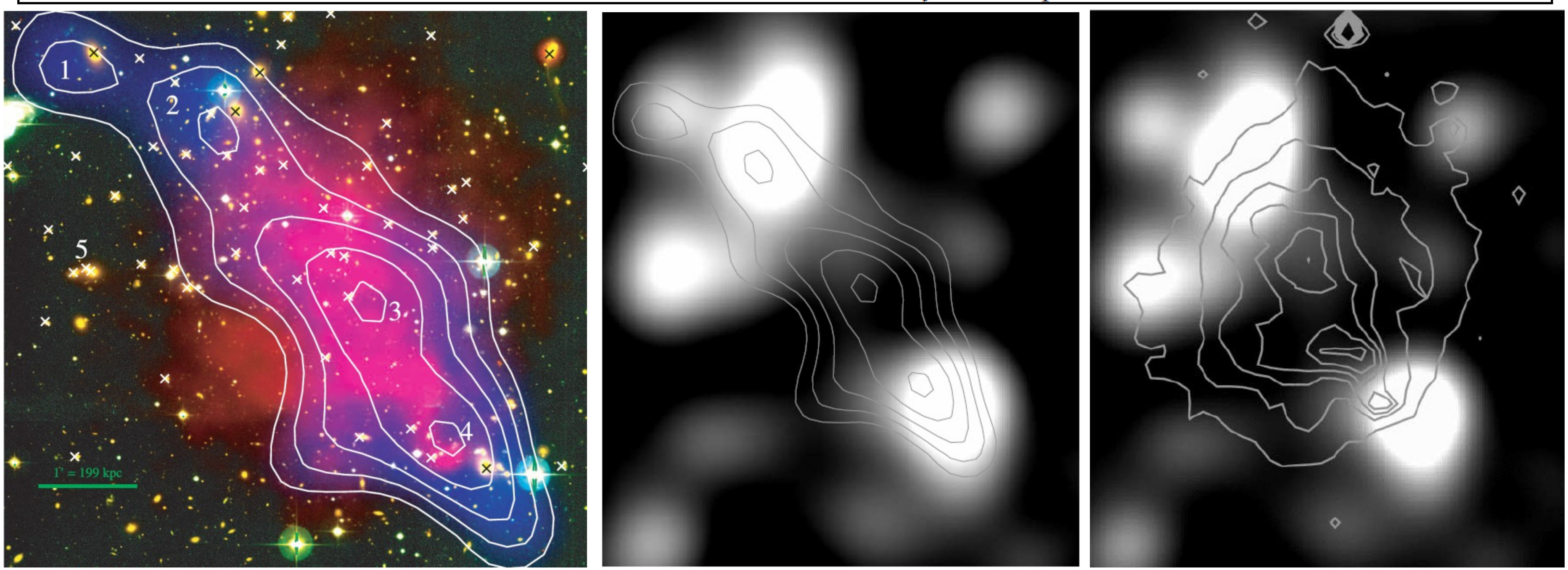

The Astrophysical Journal, 758:128 (16pp), 2012 October 20 doi:10.1088/0004-637X/758/2/128

ON DARK PEAKS AND MISSING MASS: A WEAK-LENSING MASS RECONSTRUCTION OF THE MERGING CLUSTER SYSTEM A520*,†

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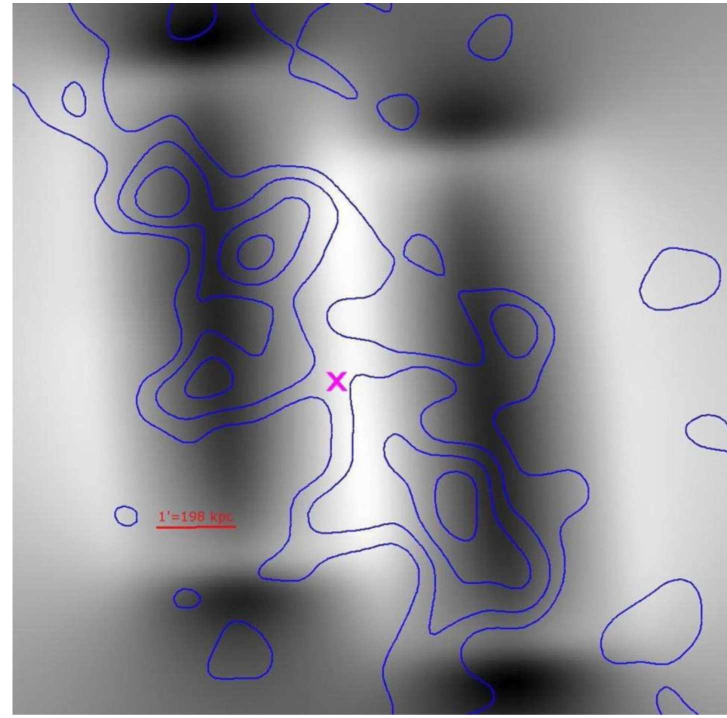
Received 2012 May 8; accepted 2012 September 4; published 2012 October 8

ABSTRACT

Merging clusters of galaxies are unique in their power to directly probe and place limits on the self-interaction cross-section of dark matter. Detailed observations of several merging clusters have shown the intracluster gas to be displaced from the centroids of dark matter and galaxy density by ram pressure, while the latter components are spatially coincident, consistent with collisionless dark matter. This has been used to place upper limits on the dark matter particle self-interaction cross-section of order 1 cm^2 g^{-1}. The cluster A520 has been seen as a possible exception. We revisit A520 presenting new *Hubble Space Telescope* Advanced Camera for Surveys mosaic images and a Magellan image set. We perform a detailed weak-lensing analysis and show that the weak-lensing mass measurements and morphologies of the core galaxy-filled structures are mostly in good agreement with previous works. There is, however, one significant difference: We do not detect the previously claimed "dark core" that contains excess mass with no significant galaxy overdensity at the location of the X-ray plasma. This peak has been suggested to be indicative of a large self-interaction cross-section for dark matter (at least $\sim 5\sigma$ larger than the upper limit of 0.7 cm^2 g^{-1} determined by observations of the Bullet Cluster). We find no such indication and instead find that the mass distribution of A520, after subtraction of the X-ray plasma mass, is in good agreement with the luminosity distribution of the cluster galaxies. We conclude that A520 shows no evidence to contradict the collisionless dark matter scenario.



The Astrophysical Journal, 783:78 (18pp), 2014 March 10 doi:10.1088/0004-637X/783/2/78

HUBBLE SPACE TELESCOPE/ADVANCED CAMERA FOR SURVEYS CONFIRMATION OF THE DARK SUBSTRUCTURE IN A520*

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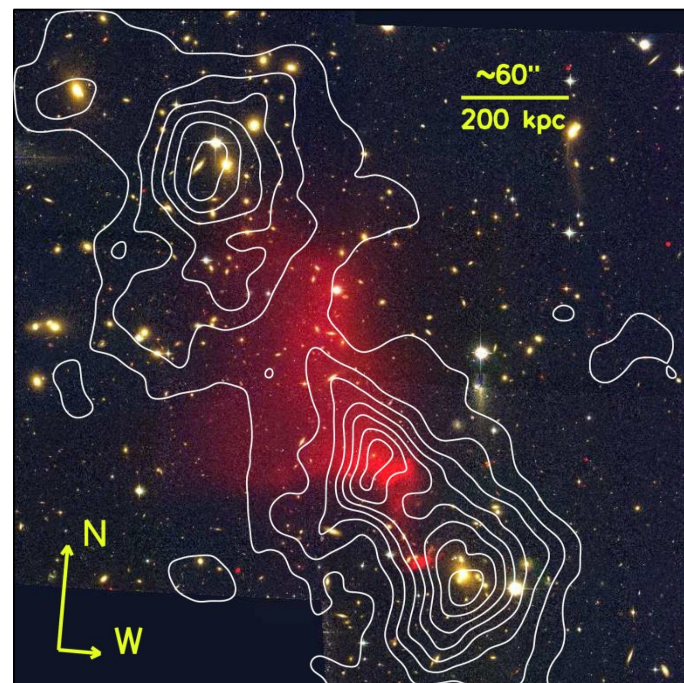
Received 2013 September 18; accepted 2014 January 15; published 2014 February 19

ABSTRACT

We present a weak-lensing study of the cluster A520 based on Advanced Camera for Surveys (ACS) data. The excellent data quality provides a mean source density of ~ 109 arcmin^{-2}, which improves both resolution and significance of the mass reconstruction compared to a previous study based on Wide Field Planetary Camera 2 (WFPC2) images. We take care in removing instrumental effects such as the charge trailing due to radiation damage of the detector and the position-dependent point-spread function. This new ACS analysis confirms the previous claims that a substantial amount of dark mass is present between two luminous subclusters where we observe very little light. The centroid of the dark peak in the current ACS analysis is offset to the southwest by $\sim 1'$ with respect to the centroid from the WFPC2 analysis. Interestingly, this new centroid is in better agreement with the location where the X-ray emission is strongest, and the mass-to-light ratio estimated with this centroid is much higher ($813 \pm 78\, M_\odot/L_{R\odot}$) than the previous value; the aperture mass with the WFPC2 centroid provides a consistent mass. Although we cannot provide a definite explanation for the dark peak, we discuss a revised scenario, wherein dark matter with a more conventional range ($\sigma_{\rm DM}/m_{\rm DM} < 1$ cm^2 g^{-1}) of self-interacting cross-section can lead to the detection of this dark substructure. If supported by detailed numerical simulations, this hypothesis opens up the possibility that the A520 system can be used to establish a lower limit of the self-interacting cross-section of dark matter.



Alternatives to dark matter: modified laws of dynamics and gravity

- Newtonian limit of GR for $\Lambda \neq 0$ (Adler, Bazin & Schiffer, 1965): $\phi = -\frac{GM}{r} - \frac{\Lambda}{6} r^2$
- Modified Newtonian Dynamics (MOND):

$$\vec{F} = m \cdot \mu\left(\frac{a}{a_0}\right)\vec{a} \qquad \mu(x)=1 \ \ if \ \ |x| \gg 1, \quad \mu(x)=x \ \ if \ \ |x| \ll 1$$

- Flat rotation velocity:

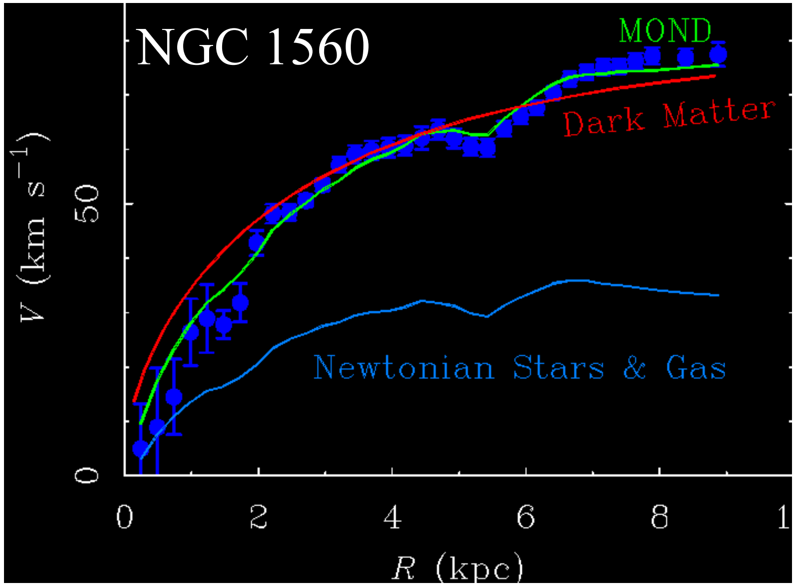
$$\frac{GMm}{r^2} = m\frac{\left(\frac{v^2}{r}\right)^2}{a_0} \Rightarrow v^4 = GMa_0$$

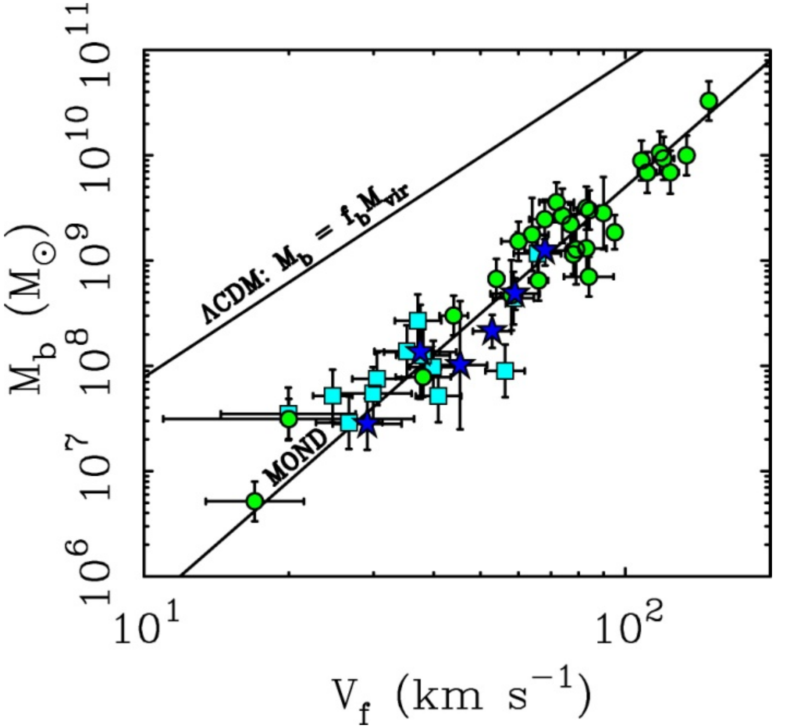
- Modified gravity (just one of many models):

$$\Phi(r) \propto \begin{cases} 1/r, & \text{at subgalactic scales} \\ \ln r, & \text{at (extra)galactic scales} \end{cases}$$

- Constant circular velocity:

$$\phi(r) \propto v_0^2 \ln r \ \Rightarrow \ v_c^2(r) = r \cdot \frac{d\phi}{dr} = v_0^2$$





Baryonic Tully-Fisher relation of gas rich galaxies (McGaugh 2011, *PRL*, 106, 121303)

The Astrophysical Journal, 718:60–65, 2010 July 20

BULLET CLUSTER: A CHALLENGE TO ΛCDM COSMOLOGY

Jounghun Lee[1] and Eiichiro Komatsu[2]

The Bullet Cluster 1E0657-558 evidence shows modified gravity in the absence of dark matter

J. R. Brownstein[1★] and J. W. Moffat[2★]

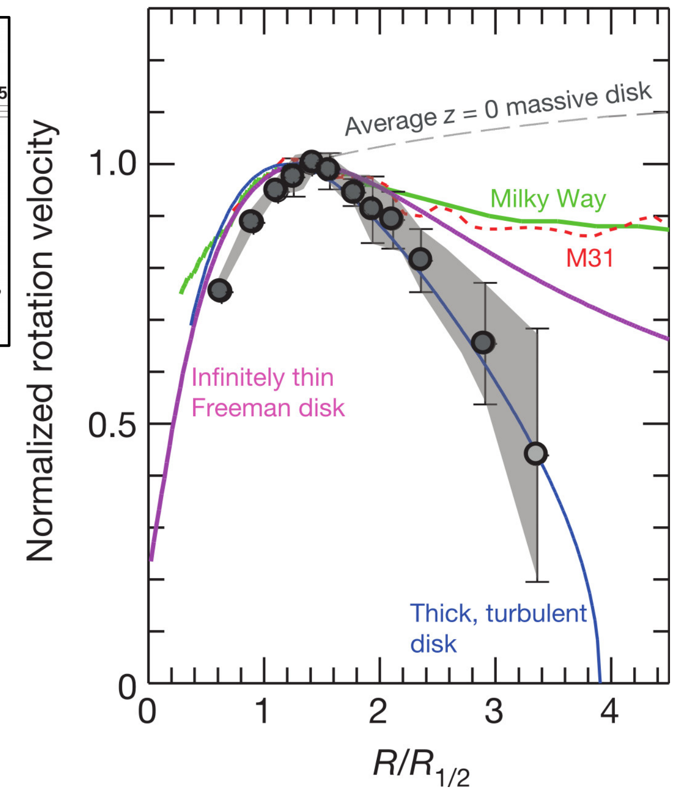
Mon. Not. R. Astron. Soc. **382,** 29–47 (2007)

LETTER

doi:10.1038/nature21685

Strongly baryon-dominated disk galaxies at the peak of galaxy formation ten billion years ago

R. Genzel[1,2], N. M. Förster Schreiber[1], H. Übler[1], P. Lang[1], T. Naab[3], R. Bender[1,4], L. J. Tacconi[1], E. Wisnioski[1], S. Wuyts[1,5], T. Alexander[6], A. Beifiori[1,4], S. Belli[1], G. Brammer[7], A. Burkert[1,3], C. M. Carollo[8], J. Chan[1], R. Davies[1], M. Fossati[1,4], A. Galametz[1,4], S. Genel[9], O. Gerhard[1], D. Lutz[1], J. T. Mendel[1,4], I. Momcheva[10], E. J. Nelson[1,10], A. Renzini[11], R. Saglia[1,4], A. Sternberg[12], S. Tacchella[8], K. Tadaki[1] & D. Wilman[1,4]



DARK MATTER sciencemag.org SCIENCE 1462 27 MARCH 2015 • VOL 347 ISSUE 6229

The nongravitational interactions of dark matter in colliding galaxy clusters

David Harvey,[1,2*] Richard Massey,[3] Thomas Kitching,[4] Andy Taylor,[2] Eric Tittley[2]

Planck 2015 results. XIV. Dark energy and modified gravity

Planck Collaboration: P. A. R. Ade[90], N. Aghanim[63], M. Arnaud[77], M. Ashdown[73,7], J. Aumont[63], C. Baccigalupi[89], A. J. Banday[101,10],

Exam question

1. Basic principles and shape distortions due to weak lensing and its application for detection of dark matter

Literature

Textbook:

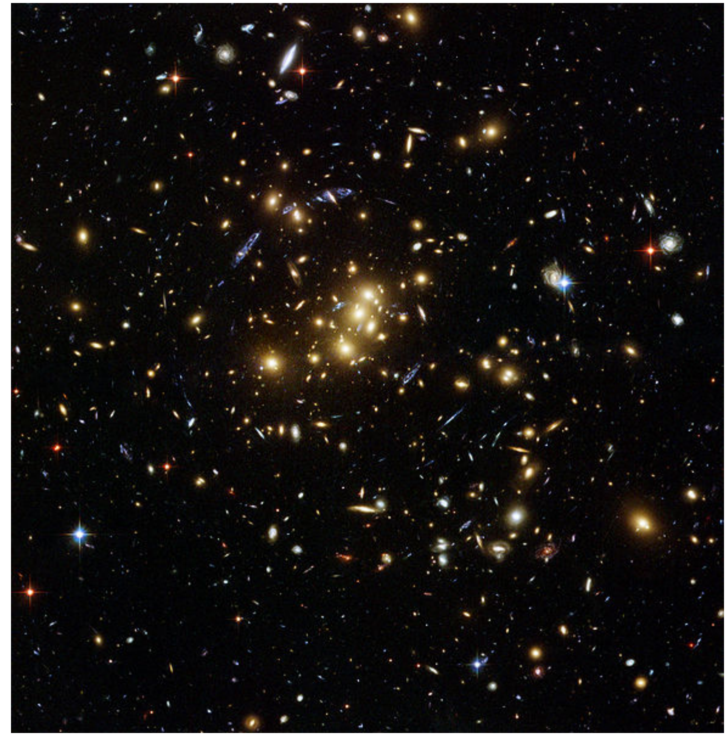
- *Gravitational Lensing*: Strong, Weak and Micro, Book Series: Saas-Fee Advanced Courses
  1. P. Schneider - *Introduction to Gravitational Lensing and Cosmology*
  2. C. S. Kochanek - *Strong Gravitational Lensing*
  3. P. Schneider - *Weak Gravitational Lensing*
  4. J. Wambsganss - *Gravitational Microlensing*

Exercise 1

Consider the cluster of galaxies CL0024+17 located in Pisces (see Fig in the right) and assume an axially-symmetric mass distribution for the cluster which can be described by the following parametric lens model with isothermal profile:

$$\kappa(\theta) = 0.5 \left(\frac{\theta}{\theta_E}\right)^{-1}, \quad |\gamma| = \kappa,$$

where Einstein radius of the cluster is $\theta_E = 0'.5$.

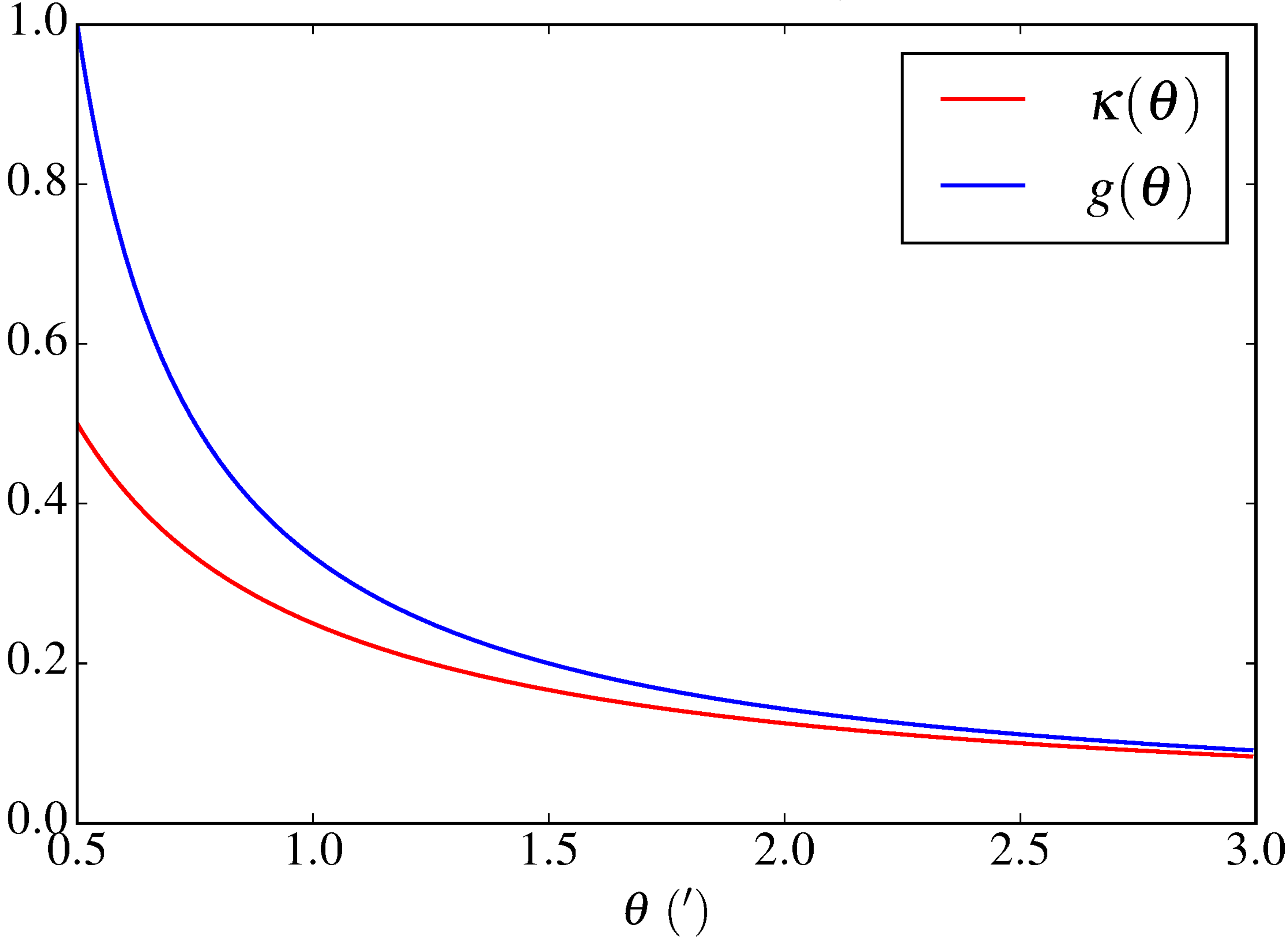


Cluster of galaxies CL0024+17

a) Plot the radial mass profile κ and reduced shear g as a function of angular separation θ from the cluster center in the range: $0'.5 < \theta < 3'$.

b) Plot the distorted images of four circular sources with radius $R = 0'.1$, located at x-axis and with the following angular separations from the cluster center: 0'.8, 1'.0, 1'.4 and 1'.8. Assume that shear direction is along x-axis.

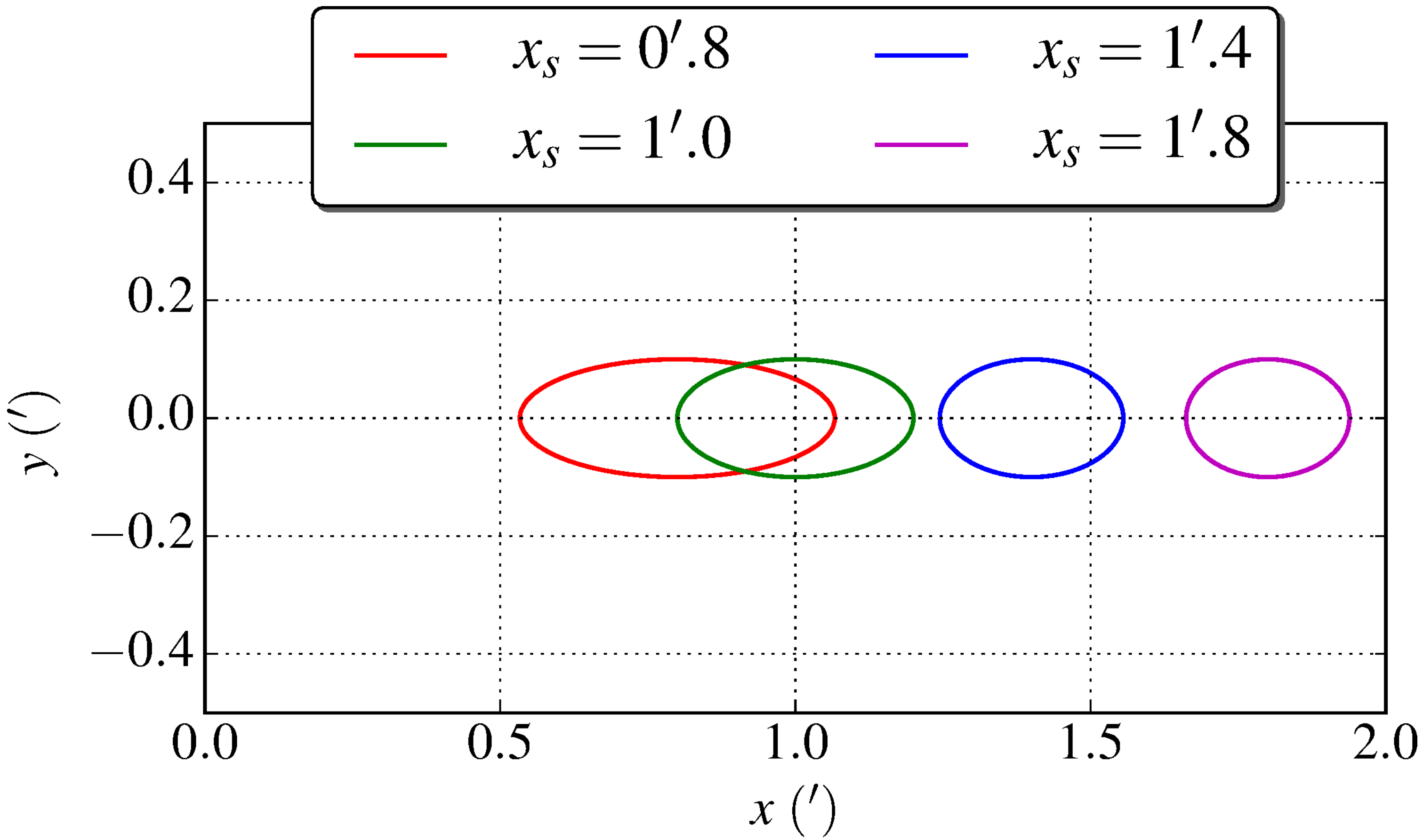
Solution 1a)



Solution is obtained by Python script "conv_shear.py"

Solution 1b)

Distorted images of circular sources are ellipses with major and minor axes: $a = \dfrac{R}{1-\kappa-\gamma} \quad b = \dfrac{R}{1-\kappa+\gamma}$



Solution is obtained by Python script "weakl_circs.py"