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GRAVITATIONAL LENSING LIMITS ON THE COSMOLOGICAL CONSTANT IN A FLAT UNIVERSE

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ABSTRACT

Inflationary cosmological theories predict and some more general aesthetic criteria suggest that the large-scale spatial curvature of the universe k should be accurately zero (i.e., flat), a condition which is satisfied when the universe's present mean density $\bar{\rho}_0$ and the value of the cosmological constant Λ have certain pairs of values. Available data on the frequency of multiple image lensing of high-redshift quasars by galaxies suggest that *the cosmological constant cannot make a dominant contribution to producing a flat universe.* In particular, if the mean density of the universe is as small as the baryon density inferred from standard cosmic nucleosynthesis calculations or as determined from typical dynamical studies of galaxies and galaxy clusters, then a value of Λ large enough to produce a $k = 0$ universe would result in a substantially higher frequency of multiple-image lensing of quasars than has been observed so far. Shortcomings of the available lens data and uncertainties concerning galaxy properties allow some possibility of escaping this conclusion, but systematic searches for gravitational lenses and continuing investigations of galaxy mass distributions should soon provide decisive information. It is also noted that nonzero curvature cosmological models can account for the observed frequency of galaxy-quasar lens systems and for a variety of other constraints.

Subject headings: cosmology — gravitational lenses — quasars

I. FLAT COSMOLOGICAL MODELS

Convention and convenience lead to a notation in which the triple of dimensional constants $\bar{\rho}_0$, Λ, and Hubble's constant H_0 are collapsed into two dimensionless constants specifying the world model

$$\Omega_0 = \frac{8\pi G\bar{\rho}_0}{3H_0^2} \tag{1}$$

and

$$\lambda = \frac{\Lambda}{3H_0^2}\,, \tag{2}$$

thus suppressing the frequently irrelevant scale factor (H_0). For the purposes of this *Letter*, attention is then directed upon those models which satisfy

$$k \propto (\Omega_0 + \lambda - 1) = 0 \tag{3}$$

and therefore

$$\Omega_0 + \lambda = 1\,. \tag{4}$$

It is worth emphasizing that failure of the universe to satisfy equation (4) would amount to contradiction of our most successful version of the big bang theory (Guth 1981; Sato 1981; Linde 1982; Albrecht and Steinhardt 1982; see, however, Penrose 1989 and Steinhardt 1990) and raise some worrisome general problems of "fine tuning" (Collins and Hawking 1973; Dicke and Peebles 1979). It is therefore desirable to test the validity of equation (4) as strongly as possible, irrespective of determining specific values for any of the three basic cosmological parameters.

II. GRAVITATIONAL LENSING PROBABILITIES

Turner, Ostriker, and Gott (1984, hereafter TOG) developed a formalism for calculating statistical quantities associated with the gravitational lensing of quasars by the (more or less) known intervening population of galaxies in $\lambda = 0$ cosmological models for Ω_0 values of 0 or 1. One particular quantity of interest was the integrated probability, referred to as a gravitational lensing optical depth $\tau_{\rm GL}(z_Q)$, of a quasar at a redshift $z_Q = y - 1$ being multiply imaged by a galaxy along the line of sight. TOG noted that the shape of this function is particularly sensitive to cosmic parameters and insensitive to the lens mass distribution. Gott, Park, and Lee (1989) provide formulae for generalizing the TOG formalism to arbitrary Robertson-Walker cosmologies. Using their comoving coordinate approach but retaining the TOG notation and basic model (galaxies represented as an unevolving population of randomly distributed singular isothermal spheres), it is then straightforward to obtain the general "filled beam" expression for $k = 0$

$$\tau_{\rm GL}(z_Q) = \frac{F}{30}\left[\int_1^y \frac{dw}{(\Omega_0 w^3 - \Omega_0 + 1)^{1/2}}\right]^3, \tag{5}$$

where

$$F = 16\pi^3 n_0\left(\frac{c}{H_0}\right)^3\left(\frac{\sigma}{c}\right)^4 \tag{6}$$

is a dimensionless parameter proportional to the gravitational lensing effectiveness of a population of singular isothermal spheres with one-dimensional velocity dispersions σ and comoving space densities n_0. Integrating over the Schechter (1976) luminosity function and using the Tully-Fisher (Aaronson, Mould, and Huchra 1980) and Faber-Jackson (1976) relations, TOG find that $F = 0.15$ for the observed galaxy population. Note that here, as in TOG, variation of Ω_0 corresponds only to a change in the background cosmology in which lensing occurs and *not to any change in the population of lensing objects* specified by the value of F. Also note that the distinction between the "filled beam" and "empty beam" approximations becomes increasingly unimportant for decreasing values of Ω_0, and thus equation (5) should be particularly reliable for λ-dominated cases.

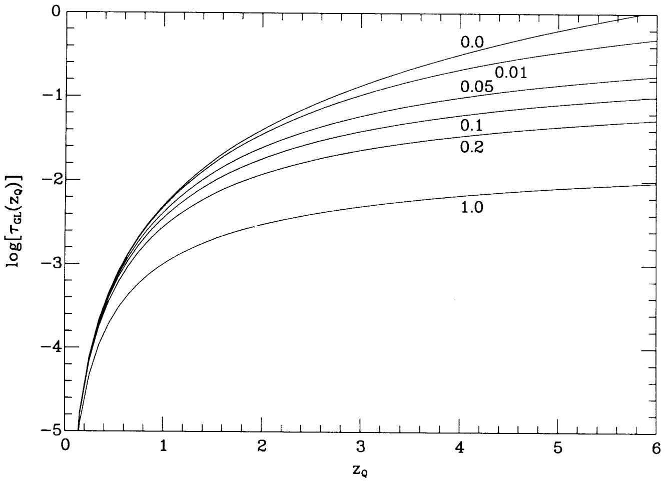

FIG. 1.—The gravitational lensing optical depth (integrated probability) for multiple imaging by intervening galaxies τ_{GL} as a function of source (e.g., quasar) redshift z_Q for six spatially flat ($k = 0$) cosmological models. The curves are labeled by their assumed values of Ω_0. Models with small values of Ω_0 and correspondingly large values of the dimensionless cosmological constant λ all show much larger values of τ_{GL} at typical quasar redshifts than a more conventional $\Omega_0 = 1$, $\lambda = 0$ model shown by the bottom curve.

The integral in equation (6) may be easily evaluated for $\Omega_0 = 1$ to recover the TOG formula

$$\tau_{GL}(z_Q) = \frac{4F}{15}\frac{(y^{1/2} - 1)^3}{y^{3/2}} \qquad (7)$$

and for $\Omega_0 = 0$ to obtain the remarkably simple result

$$\tau_{GL}(z_Q) = \frac{F}{30}(y - 1)^3 = \frac{Fz_Q^3}{30} . \qquad (8)$$

For other values of Ω_0 (and the corresponding values of λ), the integral is still trivially calculated. Figure 1 displays the function for these two limiting cases and four interesting intermediate cases: $\Omega_0 = 0.2$, 0.1, 0.05, and 0.01. These values were chosen to roughly correspond to Ω_0 values indicated by galaxy cluster and cosmic peculiar velocity studies (Trimble 1987; Oemler 1988; Dressler 1990, and references therein), galaxy rotation curves (Trimble 1987; Casertano and van Albada 1990; Sanders 1990, and references therein), and the big bang nucleosynthesis range (Yang *et al.* 1984; Pagel 1990; Fuller, Mathews, and Alcock 1988). Figure 1 shows that the optical depth for lensing by foreground galaxies is increased by at least an order-of-magnitude at high redshift in all five cosmological constant–dominated cases. This is the effect which allows a sensitive gravitational lensing test of the nature of a $k = 0$ universe.

Previous studies of the effect of nonzero λ values on gravitational lensing (Paczyński and Gorki 1981; Alcock and Anderson 1986; Gott, Park, and Lee 1989; Miralda-Escudé 1991) have concentrated on a much less sensitive quantity, the image angular separation, and thus have been unable to reach any very strong conclusions.

Very recently Fukugita, Futamase, and Kasai (1990) have independently pointed out that $\tau_{GL}(z_Q)$ is very sensitive to λ, particularly at small values of Ω_0. In addition to singular isothermal spheres, they also have treated the case of point mass lenses and have verified that the effect is rather insensitive to the lens mass distribution, a vital point. They do not, however, make any direct comparison to observational data nor, therefore, draw any conclusions with respect to favored cosmological models.

III. COMPARISON WITH OBSERVED LENSING FREQUENCIES

Equipped with equation (5) and following the procedures of TOG, it is then straightforward to calculate the expected number of multiple image gravitational lens systems (produced by the known galaxy population) to be expected in any particular quasar sample with a known distribution of redshifts. Such calculations have been carried out for three quasar samples for each of the six Ω_0 values discussed above; the results are presented in Table 1 in comparison with the observed frequency of lens systems found therein. Each of the three samples is discussed separately below.

The first sample is that of the known $z > 4$ quasars which have been studied carefully and systematically by Schneider, Schmidt, and Gunn (1989*a*, *b*, hereafter together SSG). It is of interest because these highest known redshift quasars have the greatest sensitivity of predicted lensing optical depth to Ω_0 value in a $k = 0$ universe (see Fig. 1) and because each quasar in this sample has been observed relatively carefully. Of course, the sample suffers from its very small size so that the absence of observed lens systems contradicts the predictions of only the most extreme cosmological constant–dominated models. Still, it is remarkable that even Ω_0 values as large as 0.1 ($\lambda = 0.9$) indicate a roughly 50% chance that one or more of the 11 known $z > 4$ quasars would be multiply imaged. It is clear that as the number of such very high redshift quasars known grows, the sample will provide a strong measure of λ in a $k = 0$ universe.

The second sample is the 420 confirmed quasars found in the extensive UV excess survey recently reported by Boyle *et al.* (1990, hereafter BFSP). This sample is a good one because it is large and has been selected uniformly using automated and well-defined criteria. It is probably the most statistically useful quasar sample available. It is also important that all the quasar images in this sample have been measured and characterized with the COSMOS plate scanning engine (MacGillivary and Stobie 1985). For this sample and among the $k = 0$ models

TABLE 1

GALAXY-QUASAR GRAVITATIONAL LENSING FREQUENCIES

A. OBSERVED LENSES

PARAMETER	SAMPLE NAME		
	SSG	BFSP	HB
Definition	$z > 4$	UV excess	Known quasars
Number of quasars	11	420	4250
Number of lenses	0	0	9

B. PREDICTED LENSES

Ω_0:λ:k	SAMPLE NAME		
	SSG	BFSP	HB
1.0:0.0:0	0.08	0.78	9.4
0.2:0.8:0	0.40	2.8	37
0.1:0.9:0	0.72	4.2	57
0.05:0.95:0	1.2	5.5	80
0.01:0.99:0	2.7	7.8	120
0.0:1.0:0	4.2	8.7	150
0.0:0.0:−1	0.18	1.4	17

listed in Table 1, only the $\Omega_0 = 1$ model gives a good fit, while the $\Omega_0 = 0.2$ model marginally falls within the 95% confidence interval.

Finally, the third sample is the comprehensive listing of nearly all known quasars by Hewitt and Burbidge (1987, 1989; hereafter together HB), complete through 1988 June. Its primary virtues are that it contains a very large number of quasars (4250) and that it actually contains nine gravitational lens systems judged to be either "confirmed" or "probable" in a late 1988 review of gravitational lens observations (Turner 1989). They are 0142−100, 0957+561, 1115+080, 1120+019, 1413+117, 1635+267, 2016+112, 2237+031, and 2345+007. It also contains within it about half of the first (SSG) sample but virtually none of the second (BFSP). Its main defect, for present purposes, is that it is very heterogeneously observed and thus very unevenly searched for lens systems. Nevertheless, taken at face value, this sample gives excellent agreement with a $\Omega_0 = 1$ ($\lambda = 0$) model and strongly excludes all of the nonzero λ models listed in Table 1.

The most straightforward, and probably correct in the author's view, interpretation of Table 1 is that either the universe has nonzero curvature or Ω_0 is substantially larger than indicated by dynamical studies and than the baryonic density derived from cosmic nucleosynthesis calculations. If $k = 0$, then $\Omega_0 = 1$ and $\lambda = 0$ would seem to be the favored possibility.

IV. ALTERNATE POSSIBILITIES

How might the above conclusion be avoided? There are essentially two possibilities: Either equation (5) substanially overestimates the expected number of lens systems, or the observed numbers of lens systems reported in Table 1 are incomplete due to multiple image lens systems having gone unrecognized. Both possibilities may apply to some degree, but quite large effects would be necessary to account for the magnitude of the discrepancies between the observed and small Ω_0 predicted numbers in Table 1, especially for the HB sample.

First, the value of F may have been overestimated by TOG; it is, after all, quite sensitive to galaxy characteristic velocity dispersions. However, these have been measured reasonably accurately in recent years (Davies *et al.* 1987) and are confirmed by detailed modeling of some individual lens systems (Schneider *et al.* 1988; Langston *et al.* 1989; Kochanek *et al.* 1989). Individual galaxy lensing cross sections could be substantially reduced below that implied by the singular isothermal sphere model (Dyer 1984; Hinshaw and Krauss 1987; Kochanek and Blandford 1987), but again, models of individual lens systems, accurately measured velocity dispersions, detailed dynamical studies (see Trimble 1987 and Fall 1987 for a review), and photometric studies (Kormendy 1985; Lauer 1985, 1987) suggest otherwise. (Note that 80% of the contribution to TOG's estimate of F comes from early-type galaxies, and it is their mass distributions which are most critical.) Alternately, the integral in equation (5) may be too extensive if galaxies did not exist at substantial redshifts (e.g., greater than unity); however, it is difficult to see how galaxy formation could have occurred at low redshift in a cosmological constant–dominated universe (because linear regime density perturbations are unbound at low redshifts). The mix of early- and late-type galaxies adopted by TOG could also be somewhat in error with moderate consequences for F. If all such uncertainties are taken into account and cooperate to reduce the value of F, a reduction by more than a factor of 2 is possible, although changes of order 50% appear considerably more likely. [Note that F is independent of H_0 because the combination $n_0 H_0^{-3}$ can be determined directly from galaxy number-magnitude counts (Turner and Gott 1976; Efstathiou, Ellis, and Peterson 1988).]

Second, on the observational side, it is certainly possible, even likely, that some lens systems have been missed among known quasars, particularly on the HB list, but it is difficult to imagine that the incompleteness could be a large factor (e.g., that 75% of the lens systems have been missed). It is important to recognize in this regard that the large majority of the quasars on the HB list and all of those in the other two samples were discovered in the 1980s when observers were well aware of the possibility and importance of gravitational lensing. Moreover, several investigators have carried out active programs (Surdej *et al.* 1988; Meylan and Djorgovski 1989; Yee 1990; Schneider 1990; Burke *et al.* 1990) searching for lenses among known quasars (particularly those at high redshift) by high-resolution (sub–arc second) imaging without uncovering any large number of them.

It is also important to note that there is another effect which, if properly taken into account, would increase (perhaps greatly increase) the predicted number of lens systems for the various Table 1 models. It is the "amplification bias" (see TOG), the selection bias by which lens systems are preferentially included in magnitude-limited samples due to the lensing-induced increase in their flux. This bias can be quite strong under many circumstances (Turner 1980; TOG; Ostriker and Vietri 1986; Schneider 1987) and could easily be larger than all of the contravening effects discussed in the previous paragraphs. In this sense, given the adopted value of F, the predicted numbers of lens systems in Table 1 should be regarded as *lower limits* to the actually expected numbers in magnitude-limited samples.

On balance, it would not be at all surprising if the detailed numbers given in Table 1 require significant adjustment to take account of the various effects discussed above, but given the magnitude of the discrepancies, it does not appear easy to avoid their $k \neq 0$ or Ω_0 large implications.

In any case, it should be straightforward to check or close the various possible loopholes discussed above. Both systematic searches for gravitational lens systems and investigations of galaxy mass distributions are already being pursued for a variety of reasons and with very substantial observational resources. Whatever the ultimate resolution of the complexities of characterizing the galaxy population and accounting for selection biases, the strong dependence of $\tau_{\rm GL}(z_Q)$ on λ in a $k = 0$ universe (displayed in Fig. 1) makes it a very promising cosmological diagnostic.

V. CONCLUSIONS AND DISCUSSION

The basic conclusion obtained above may be simply stated: Barring either a surprisingly large number of undiscovered multiple image lens systems among known quasars or a surprisingly large error in our understanding of the properties of known galaxies (or some conspiracy of the two), the theoretical prediction of a spatially flat universe cannot be accommodated by appeal to λ values near unity.

Although this *Letter* has restricted its attention to $k = 0$ models, it should be clearly understood that nonzero curvature models (e.g., $\lambda = 0$, small Ω_0 cases) are not only consistent with the observed frequency of galaxy-quasar lens systems (see the last row of Table 1) but also are more easily able to account for various other constraints. These include the ages of the oldest stars (Twarog 1980; Sandage 1982) and heavy elements (Fowler 1989; Clayton 1989), the observed abundances of the light elements (Pagel 1990), the dynamical masses of galaxies and galaxy clusters (Trimble 1987; Oemler 1988), the existence of high-luminosity quasars at high redshifts (SSG; Turner 1991), deep galaxy counts (Tyson 1988; Fukugita *et al.* 1990), and the absence (so far) of detected elementary particles capable of accounting for a high-density universe (for a review, see Spergel 1989). It is thus reasonable to suspect that the universe may (again) be failing to conform to attractive theoretical concepts.

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